AP Physics C
Summer Assignment
Mr. Peterson

Welcome to AP Physics C! It's great that you are interested in this subject. In AP Physics C, we'll be examining the fundamentals of physics. This course covers the first year of college physics and is designed for you to develop a deep understanding of physics and to prepare you for superior performance on the AP test.

This course will require you to commit to working on physics every single day. To get you started, and to make sure you are interested and really want to put the work into this comprehensive and challenging topic, I've assigned some tasks for you to complete this summer, worth 100 points. Please show all the steps in your work. If insufficient work is shown because it was all done on the calculator, then your calculator will receive half the credit!

You will need to turn it in on the first day of school. It will be worth a substantial grade, and part of what will determine if AP Physics C is right for you. The other part of that consideration is a **pre-test** that we'll be taking during the first week of school, which covers the summer assignment. Please email me at **mpeterson@tmsacademy.org** if you have any questions!

Summer Assignments:

I. Purchase an AP Physics C Review Book.

II. Purchase a graphing calculator. You will not be allowed to run to the math classrooms to get one or allowed to use your phone during class.

III. Purchase a lab notebook.

We have covered most of this chapter in AP Physics 1. So, it will not be a big stretch. Do not worry too much about the last section. It has integration, which we will cover at the beginning of the year.

IV. In the lab notebook, write the chapter 5 summary**.**

The following work is to be grouped by diagrams, equations, and finally examples. If it is not in the lab book in this way, you will be required to rewrite it.

V. Next, in the lab notebook, draw all the diagrams for each section. Take notes to make sure you understand them.

VI. Then, for each section, go through each section and derive all the equations.

VII. Following this, you need to go through the sections and do all the examples.

VIII. Finally, review all the diagrams, derivations, and examples so that you can replicate them for the first assessment.

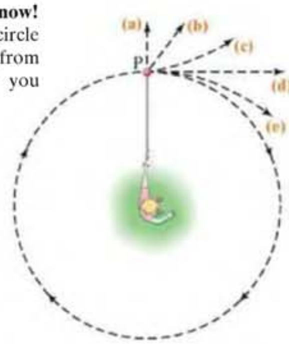
Newton's laws are fundamental in physics. These photos show two situations of using Newton's laws which involve some new elements in addition to those discussed in the previous Chapter. The downhill skier illustrates *friction* on an incline, although at this moment she is not touching the snow, and so is retarded only by air resistance which is a velocity-dependent force (an optional topic in this Chapter). The people on the rotating amusement park ride below illustrate the dynamics of circular motion.

CHAPTER 5

Using Newton's Laws: Friction, Circular Motion, Drag Forces

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!
You revolve a ball around you in a horizontal circle at constant speed on a string, as shown here from above. Which path will the ball follow if you let go of the string at point P?



This chapter continues our study of Newton's laws and emphasizes their fundamental importance in physics. We cover some important applications of Newton's laws, including friction and circular motion. Although some material in this Chapter may seem to repeat topics covered in Chapter 4, in fact, new elements are involved.

5–1 Applications of Newton's Laws Involving Friction

Until now we have ignored friction, but it must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 5–1. When we try to slide an object across another surface, these microscopic bumps impede the motion. Exactly what is happening at the microscopic level is not yet fully understood. It is thought that the atoms on a bump of one surface may come so close to the atoms of the other surface that attractive electric forces between the atoms could "bond" as a tiny weld between the two surfaces. Sliding an object across a surface is often jerky, perhaps due to the making and breaking of these bonds. Even when a round object rolls across a surface, there is still some friction, called *rolling friction*, although it is generally much less than when objects slide across a surface. We focus our attention now on sliding friction, which is usually called **kinetic friction** (*kinetic* is from the Greek for "moving").

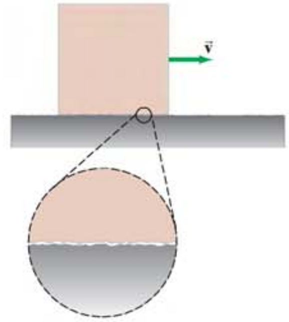
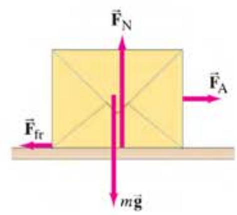


FIGURE 5–1 An object moving to the right on a table or floor. The two surfaces in contact are rough, at least on a microscopic scale.

When an object slides along a rough surface, the force of kinetic friction acts opposite to the direction of the object's velocity. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, experiment shows that the friction force is approximately proportional to the *normal force* between the two surfaces, which is the force that either object exerts on the other and is perpendicular to their common surface of contact (see Fig. 5–2). The force of friction between hard surfaces in many cases depends very little on the total surface area of contact; that is, the friction force on this book is roughly the same whether it is being slid on its wide face or on its spine, assuming the surfaces have the same smoothness. We consider a simple model of friction in which we make this assumption that the friction force is independent of area. Then we write the proportionality between the magnitudes of the friction force F_{fr} and the normal force F_N as an equation by inserting a constant of proportionality, μ_k:

$$F_{fr} = \mu_k F_N. \qquad \text{[kinetic friction]}$$

FIGURE 5–2 When an object is pulled along a surface by an applied force ($\vec{F}_A$), the force of friction $\vec{F}_{fr}$ opposes the motion. The magnitude of $\vec{F}_{fr}$ is proportional to the magnitude of the normal force (F_N).



This relation is not a fundamental law; it is an experimental relation between the magnitude of the friction force F_{fr}, which acts parallel to the two surfaces, and the magnitude of the normal force F_N, which acts perpendicular to the surfaces. It is *not* a vector equation since the two forces have directions perpendicular to one another. The term μ_k is called the *coefficient of kinetic friction*, and its value depends on the nature of the two surfaces. Measured values for a variety of surfaces are given in Table 5–1. These are only approximate, however, since μ depends on whether the surfaces are wet or dry, on how much they have been sanded or rubbed, if any burrs remain, and other such factors. But μ_k is roughly independent of the sliding speed, as well as the area in contact.

TABLE 5–1 Coefficients of Friction[†]

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

[†]Values are approximate and intended only as a guide.

What we have been discussing up to now is *kinetic friction*, when one object slides over another. There is also **static friction**, which refers to a force parallel to the two surfaces that can arise even when they are not sliding. Suppose an object such as a desk is resting on a horizontal floor. If no horizontal force is exerted on the desk, there also is no friction force. But now suppose you try to push the desk, and it doesn't move. You are exerting a horizontal force, but the desk isn't moving, so there must be another force on the desk keeping it from moving (the net force is zero on an object at rest). This is the force of *static friction* exerted by the floor on the desk. If you push with a greater force without moving the desk, the force of static friction also has increased. If you push hard enough, the desk will eventually start to move, and kinetic friction takes over. At this point, you have exceeded the maximum force of static friction, which is given by $(F_{fr})_{max} = \mu_s F_N$, where μ_s is the *coefficient of static friction* (Table 5–1). Because the force of static friction can vary from zero to this maximum value, we write

$$F_{fr} \leq \mu_s F_N. \qquad \text{[static friction]}$$

You may have noticed that it is often easier to keep a heavy object sliding than it is to start it sliding in the first place. This is consistent with μ_s generally being greater than μ_k (see Table 5–1).

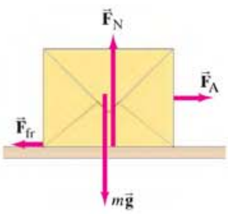


FIGURE 5–2 Repeated for Example 5–1.

EXAMPLE 5–1 **Friction: static and kinetic.** Our 10.0-kg mystery box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal external applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

APPROACH We don't know, right off, if we are dealing with static friction or kinetic friction, nor if the box remains at rest or accelerates. We need to draw a free-body diagram, and then determine in each case whether or not the box will move: the box starts moving if F_A is greater than the maximum static friction force (Newton's second law). The forces on the box are gravity $m\vec{g}$, the normal force exerted by the floor $\vec{F}_N$, the horizontal applied force $\vec{F}_A$, and the friction force $\vec{F}_{fr}$, as shown in Fig. 5–2.

SOLUTION The free-body diagram of the box is shown in Fig. 5–2. In the vertical direction there is no motion, so Newton's second law in the vertical direction gives $\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0$. Hence the normal force is

$$F_N = mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}.$$

(*a*) Because $F_A = 0$ in this first case, the box doesn't move, and $F_{fr} = 0$.
(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98.0\,\text{N}) = 39\,\text{N}.$$

When the applied force is $F_A = 10\,\text{N}$, the box will not move. Newton's second law gives $\Sigma F_x = F_A - F_{fr} = 0$, so $F_{fr} = 10\,\text{N}$.
(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{fr} = 20\,\text{N}$ to balance the applied force.
(*d*) The applied force of 38 N is still not quite large enough to move the box; so the friction force has now increased to 38 N to keep the box at rest.
(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

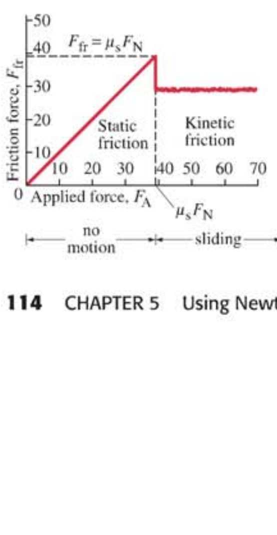
$$F_{fr} = \mu_k F_N = (0.30)(98.0\,\text{N}) = 29\,\text{N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\,\text{N} - 29\,\text{N} = 11\,\text{N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2$$

as long as the applied force is 40 N. Figure 5–3 shows a graph that summarizes this Example.

FIGURE 5–3 Example 5–1. Magnitude of the force of friction as a function of the external force applied to an object initially at rest. As the applied force is increased in magnitude, the force of static friction increases linearly to just match it, until the applied force equals $\mu_s F_N$. If the applied force increases further, the object will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.



Friction can be a hindrance. It slows down moving objects and causes heating and binding of moving parts in machinery. Friction can be reduced by using lubricants such as oil. More effective in reducing friction between two surfaces is to maintain a layer of air or other gas between them. Devices using this concept, which is not practical for most situations, include air tracks and air tables in which the layer of air is maintained by forcing air through many tiny holes. Another technique to maintain the air layer is to suspend objects in air using magnetic fields ("magnetic levitation"). On the other hand, friction can be helpful. Our ability to walk depends on friction between the soles of our shoes (or feet) and the ground. (Walking involves static friction, not kinetic friction. Why?) The movement of a car, and also its stability, depend on friction. When friction is low, such as on ice, safe walking or driving becomes difficult.

CONCEPTUAL EXAMPLE 5–2 **A box against a wall.** You can hold a box against a rough wall (Fig. 5–4) and prevent it from slipping down by pressing hard horizontally. How does the application of a horizontal force keep an object from moving vertically?

RESPONSE This won't work well if the wall is slippery. You need friction. Even then, if you don't press hard enough, the box will slip. The horizontal force you apply produces a normal force on the box exerted by the wall (net force horizontally is zero since box doesn't move horizontally.) The force of gravity mg, acting downward on the box, can now be balanced by an upward static friction force whose maximum magnitude is proportional to the normal force. The harder you push, the greater F_N is and the greater F_{fr} can be. If you don't press hard enough, then $mg > \mu_s F_N$ and the box begins to slide down.

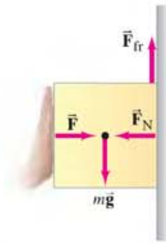


FIGURE 5–4 Example 5–2.

EXERCISE A If $\mu_s = 0.40$ and $mg = 20\,N$, what minimum force F will keep the box from falling: (*a*) 100 N; (*b*) 80 N; (*c*) 50 N; (*d*) 20 N; (*e*) 8 N?

EXAMPLE 5–3 **Pulling against friction.** A 10.0-kg box is pulled along a horizontal surface by a force F_P of 40.0 N applied at a 30.0° angle above horizontal. This is like Example 4–11 except now there is friction, and we assume a coefficient of kinetic friction of 0.30. Calculate the acceleration.

APPROACH The free-body diagram is shown in Fig. 5–5. It is much like that in Fig. 4–21, but with one more force, that of friction.

SOLUTION The calculation for the vertical (y) direction is just the same as in Example 4–11, $mg = (10.0\,kg)(9.80\,m/s^2) = 98.0\,N$ and $F_{Py} = (40.0\,N)(\sin 30.0°) = 20.0\,N$. With y positive upward and $a_y = 0$, we have

$$F_N - mg + F_{Py} = ma_y$$
$$F_N - 98.0\,N + 20.0\,N = 0,$$

so the normal force is $F_N = 78.0\,N$. Now we apply Newton's second law for the horizontal (x) direction (positive to the right), and include the friction force:

$$F_{Px} - F_{fr} = ma_x.$$

The friction force is kinetic as long as $F_{fr} = \mu_k F_N$ is less than $F_{Px} = (40.0\,N)\cos 30.0° = 34.6\,N$, which it is:

$$F_{fr} = \mu_k F_N = (0.30)(78.0\,N) = 23.4\,N.$$

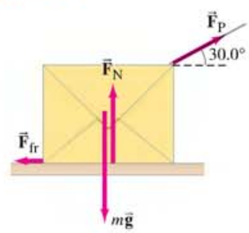
Hence the box does accelerate:

$$a_x = \frac{F_{Px} - F_{fr}}{m} = \frac{34.6\,N - 23.4\,N}{10.0\,kg} = 1.1\,m/s^2.$$

In the absence of friction, as we saw in Example 4–11, the acceleration would be much greater than this.

NOTE Our final answer has only two significant figures because our least significant input value $(\mu_k = 0.30)$ has two.

FIGURE 5–5 Example 5–3.



EXERCISE B If $\mu_k F_N$ were greater than F_{Px}, what would you conclude?

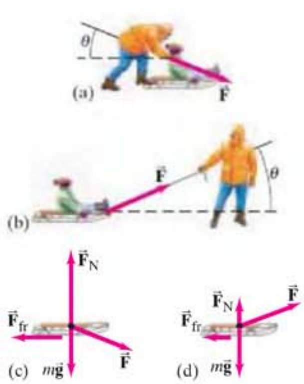
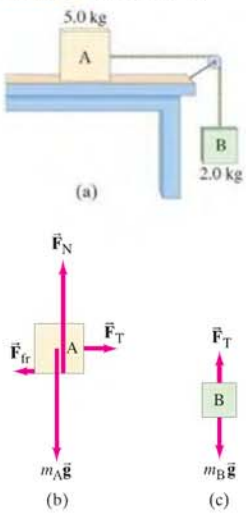


FIGURE 5–6 Example 5–4.

FIGURE 5–7 Example 5–5.



CONCEPTUAL EXAMPLE 5–4 **To push or to pull a sled?** Your little sister wants a ride on her sled. If you are on flat ground, will you exert less force if you push her or pull her? See Figs. 5–6a and b. Assume the same angle θ in each case.

RESPONSE Let us draw free-body diagrams for the sled–sister combination, as shown in Figs. 5–6c and d. They show, for the two cases, the forces exerted by you, $\vec{\mathbf{F}}$ (an unknown), by the snow, $\vec{\mathbf{F}}_N$ and $\vec{\mathbf{F}}_{fr}$, and gravity $m\vec{\mathbf{g}}$. (*a*) If you push her, and $\theta > 0$, there is a vertically downward component to your force. Hence the normal force upward exerted by the ground (Fig. 5–6c) will be larger than mg (where m is the mass of sister plus sled). (*b*) If you pull her, your force has a vertically upward component, so the normal force F_N will be less than mg, Fig. 5–6d. Because the friction force is proportional to the normal force, F_{fr} will be less if you pull her. So you exert less force if you pull her.

EXAMPLE 5–5 **Two boxes and a pulley.** In Fig. 5–7a, two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A and the table is 0.20. We ignore the mass of the cord and pulley and any friction in the pulley, which means we can assume that a force applied to one end of the cord will have the same magnitude at the other end. We wish to find the acceleration, a, of the system, which will have the same magnitude for both boxes assuming the cord doesn't stretch. As box B moves down, box A moves to the right.

APPROACH The free-body diagrams for each box are shown in Figs. 5–7b and c. The forces on box A are the pulling force of the cord F_T, gravity $m_A g$, the normal force exerted by the table F_N, and a friction force exerted by the table F_{fr}; the forces on box B are gravity $m_B g$, and the cord pulling up, F_T.

SOLUTION Box A does not move vertically, so Newton's second law tells us the normal force just balances the weight,

$$F_N = m_A g = (5.0\,\text{kg})(9.8\,\text{m/s}^2) = 49\,\text{N}.$$

In the horizontal direction, there are two forces on box A (Fig. 5–7b): F_T, the tension in the cord (whose value we don't know), and the force of friction

$$F_{fr} = \mu_k F_N = (0.20)(49\,\text{N}) = 9.8\,\text{N}.$$

The horizontal acceleration is what we wish to find; we use Newton's second law in the x direction, $\Sigma F_{Ax} = m_A a_x$, which becomes (taking the positive direction to the right and setting $a_{Ax} = a$):

$$\Sigma F_{Ax} = F_T - F_{fr} = m_A a. \qquad \text{[box A]}$$

Next consider box B. The force of gravity $m_B g = (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 19.6\,\text{N}$ pulls downward; and the cord pulls upward with a force F_T. So we can write Newton's second law for box B (taking the downward direction as positive):

$$\Sigma F_{By} = m_B g - F_T = m_B a. \qquad \text{[box B]}$$

[Notice that if $a \neq 0$, then F_T is not equal to $m_B g$.]

We have two unknowns, a and F_T, and we also have two equations. We solve the box A equation for F_T:

$$F_T = F_{fr} + m_A a,$$

and substitute this into the box B equation:

$$m_B g - F_{fr} - m_A a = m_B a.$$

Now we solve for a and put in numerical values:

$$a = \frac{m_B g - F_{fr}}{m_A + m_B} = \frac{19.6\,\text{N} - 9.8\,\text{N}}{5.0\,\text{kg} + 2.0\,\text{kg}} = 1.4\,\text{m/s}^2,$$

which is the acceleration of box A to the right, and of box B down.

If we wish, we can calculate F_T using the third equation up from here:

$$F_T = F_{fr} + m_A a = 9.8\,\text{N} + (5.0\,\text{kg})(1.4\,\text{m/s}^2) = 17\,\text{N}.$$

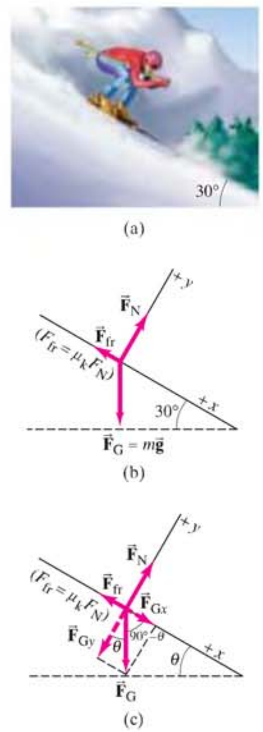
NOTE Box B is not in free fall. It does not fall at $a = g$ because an additional force, F_T, is acting upward on it.

In Chapter 4 we examined motion on ramps and inclines, and saw that it is usually an advantage to choose the x axis along the plane, in the direction of acceleration. There we ignored friction, but now we take it into account.

EXAMPLE 5–6 **The skier.** The skier in Fig. 5–8a is descending a 30° slope, at constant speed. What can you say about the coefficient of kinetic friction μ_k?

PHYSICS APPLIED
Skiing

FIGURE 5–8 Example 5–6. A skier descending a slope; $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$ is the force of gravity (weight) on the skier.



APPROACH We choose the x axis along the slope, positive pointing downslope in the direction of the skier's motion. The y axis is perpendicular to the surface as shown in Fig. 5–8b, which is the free-body diagram for our system which we choose as the skier and her skis (total mass m). The forces acting are gravity, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$, which points vertically downward (*not* perpendicular to the slope), and the two forces exerted on her skis by the snow—the normal force perpendicular to the snowy slope (*not* vertical), and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 5–8b, for convenience.

SOLUTION We have to resolve only one vector into components, the weight $\vec{\mathbf{F}}_G$, and its components are shown as dashed lines in Fig. 5–8c:

$$F_{Gx} = mg\sin\theta,$$
$$F_{Gy} = -mg\cos\theta,$$

where we have stayed general by using θ rather than 30° for now. There is no acceleration, so Newton's second law applied to the x and y components gives

$$\Sigma F_y = F_N - mg\cos\theta = ma_y = 0$$
$$\Sigma F_x = mg\sin\theta - \mu_k F_N = ma_x = 0.$$

From the first equation, we have $F_N = mg\cos\theta$. We substitute this into the second equation:

$$mg\sin\theta - \mu_k(mg\cos\theta) = 0.$$

Now we solve for μ_k:

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

which for $\theta = 30°$ is

$$\mu_k = \tan\theta = \tan 30° = 0.58.$$

Notice that we could use the equation

$$\mu_k = \tan\theta$$

to determine μ_k under a variety of conditions. All we need to do is observe at what slope angle the skier descends at constant speed. Here is another reason why it is often useful to plug in numbers only at the end: we obtained a general result useful for other situations as well.

CAUTION
Directions of gravity and the normal force

In problems involving a slope or "inclined plane," avoid making errors in the directions of the normal force and gravity. The normal force is *not* vertical: it is perpendicular to the slope or plane. And gravity is *not* perpendicular to the slope—gravity acts vertically downward toward the center of the Earth.

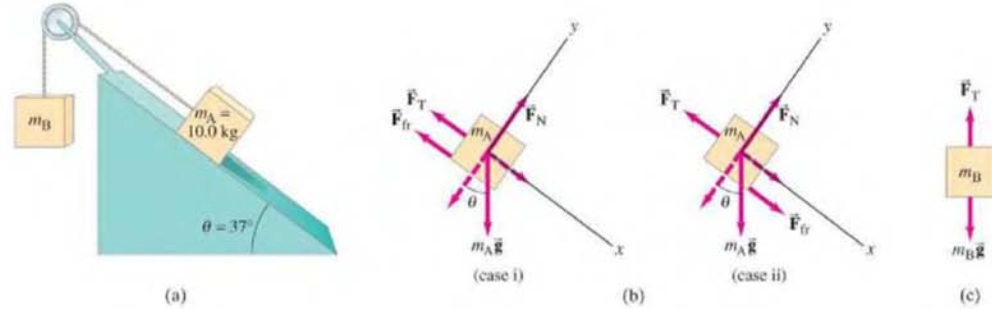


FIGURE 5–9 Example 5–7. Note choice of x and y axes.

EXAMPLE 5–7 **A ramp, a pulley, and two boxes.** A box of mass $m_A = 10.0$ kg rests on a surface inclined at $\theta = 37°$ to the horizontal. It is connected by a lightweight cord, which passes over a massless and frictionless pulley, to a second box of mass m_B, which hangs freely as shown in Fig. 5–9a. (*a*) If the coefficient of static friction is $\mu_s = 0.40$, determine what range of values for mass m_B will keep the system at rest. (*b*) If the coefficient of kinetic friction is $\mu_k = 0.30$, and $m_B = 10.0$ kg, determine the acceleration of the system.

APPROACH Figure 5–9b shows two free-body diagrams for box m_A because the force of friction can be either up or down the slope, depending on which direction the box slides: (i) if $m_B = 0$ or is sufficiently small, m_A would tend to slide down the incline, so $\vec{F}_{fr}$ would be directed up the incline; (ii) if m_B is large enough, m_A will tend to be pulled up the plane, so $\vec{F}_{fr}$ would point down the plane. The tension force exerted by the cord is labeled $\vec{F}_T$.

SOLUTION (*a*) For both cases (i) and (ii), Newton's second law for the y direction (perpendicular to the plane) is the same:

$$F_N - m_A g \cos\theta = m_A a_y = 0$$

since there is no y motion. So

$$F_N = m_A g \cos\theta.$$

Now for the x motion. We consider case (i) first for which $\Sigma F = ma$ gives

$$m_A g \sin\theta - F_T - F_{fr} = m_A a_x.$$

We want $a_x = 0$ and we solve for F_T since F_T is related to m_B (whose value we are seeking) by $F_T = m_B g$ (see Fig. 5–9c). Thus

$$m_A g \sin\theta - F_{fr} = F_T = m_B g.$$

We solve this for m_B and set F_{fr} at its maximum value $\mu_s F_N = \mu_s m_A g \cos\theta$ to find the minimum value that m_B can have to prevent motion $(a_x = 0)$:

$$\begin{aligned} m_B &= m_A \sin\theta - \mu_s m_A \cos\theta \\ &= (10.0\text{ kg})(\sin 37° - 0.40\cos 37°) = 2.8\text{ kg}. \end{aligned}$$

Thus if $m_B < 2.8$ kg, then box A will slide down the incline.

Now for case (ii) in Fig. 5–9b, box A being pulled *up* the incline. Newton's second law is

$$m_A g \sin\theta + F_{fr} - F_T = m_A a_x = 0.$$

Then the maximum value m_B can have without causing acceleration is given by

$$F_T = m_B g = m_A g \sin\theta + \mu_s m_A g \cos\theta$$

or

$$m_B = m_A \sin\theta + \mu_s m_A \cos\theta$$
$$= (10.0\,\text{kg})(\sin 37^\circ + 0.40\cos 37^\circ) = 9.2\,\text{kg}.$$

Thus, to prevent motion, we have the condition

$$2.8\,\text{kg} < m_B < 9.2\,\text{kg}.$$

(*b*) If $m_B = 10.0\,\text{kg}$ and $\mu_k = 0.30$, then m_B will fall and m_A will rise up the plane (case ii). To find their acceleration a, we use $\Sigma F = ma$ for box A:

$$m_A a = F_T - m_A g \sin\theta - \mu_k F_N.$$

Since m_B accelerates downward, Newton's second law for box B (Fig. 5–9c) tells us $m_B a = m_B g - F_T$, or $F_T = m_B g - m_B a$, and we substitute this into the equation above:

$$m_A a = m_B g - m_B a - m_A g \sin\theta - \mu_k F_N.$$

We solve for the acceleration a and substitute $F_N = m_A g \cos\theta$, and then $m_A = m_B = 10.0\,\text{kg}$, to find

$$a = \frac{m_B g - m_A g \sin\theta - \mu_k m_A g \cos\theta}{m_A + m_B}$$
$$= \frac{(10.0\,\text{kg})(9.80\,\text{m/s}^2)(1 - \sin 37^\circ - 0.30\cos 37^\circ)}{20.0\,\text{kg}}$$
$$= 0.079g = 0.78\,\text{m/s}^2.$$

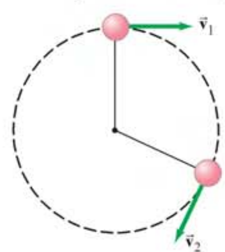
NOTE It is worth comparing this equation for acceleration a with that obtained in Example 5–5: if here we let $\theta = 0$, the plane is horizontal as in Example 5–5, and we obtain $a = (m_B g - \mu_k m_A g)/(m_A + m_B)$ just as in Example 5–5.

5–2 Uniform Circular Motion—Kinematics

An object moves in a straight line if the net force on it acts in the direction of motion, or the net force is zero. If the net force acts at an angle to the direction of motion at any moment, then the object moves in a curved path. An example of the latter is projectile motion, which we discussed in Chapter 3. Another important case is that of an object moving in a circle, such as a ball at the end of a string revolving around one's head, or the nearly circular motion of the Moon about the Earth.

An object that moves in a circle at constant speed v is said to experience **uniform circular motion**. The *magnitude* of the velocity remains constant in this case, but the *direction* of the velocity continuously changes as the object moves around the circle (Fig. 5–10). Because acceleration is defined as the rate of change of velocity, a change in direction of velocity constitutes an acceleration, just as a change in magnitude of velocity does. Thus, an object revolving in a circle is continuously accelerating, even when the speed remains constant ($v_1 = v_2 = v$). We now investigate this acceleration quantitatively.

FIGURE 5–10 A small object moving in a circle, showing how the velocity changes. At each point, the instantaneous velocity is in a direction tangent to the circular path.



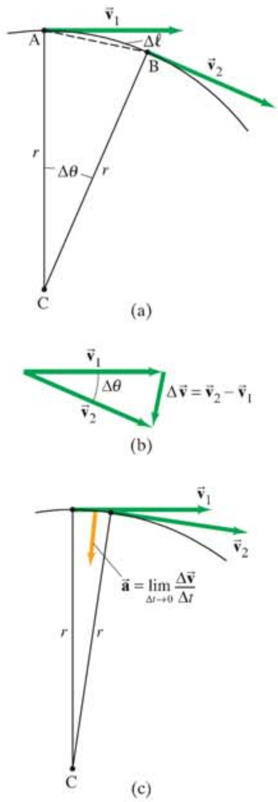
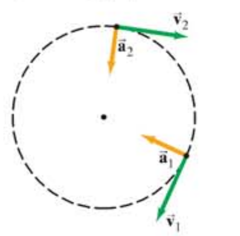


FIGURE 5–11 Determining the change in velocity, $\Delta\vec{\mathbf{v}}$, for a particle moving in a circle. The length $\Delta\ell$ is the distance along the arc, from A to B.

CAUTION

In uniform circular motion, the speed is constant, but the acceleration is not zero

FIGURE 5–12 For uniform circular motion, $\vec{\mathbf{a}}$ is always perpendicular to $\vec{\mathbf{v}}$.



Acceleration is defined as

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt},$$

where $\Delta\vec{\mathbf{v}}$ is the change in velocity during the short time interval Δt. We will eventually consider the situation in which Δt approaches zero and thus obtain the instantaneous acceleration. But for purposes of making a clear drawing (Fig. 5–11), we consider a nonzero time interval. During the time interval Δt, the particle in Fig. 5–11a moves from point A to point B, covering a distance $\Delta\ell$ *along the arc* which subtends an angle $\Delta\theta$. The change in the velocity vector is $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = \Delta\vec{\mathbf{v}}$, and is shown in Fig. 5–11b.

Now we let Δt be very small, approaching zero. Then $\Delta\ell$ and $\Delta\theta$ are also very small, and $\vec{\mathbf{v}}_2$ will be almost parallel to $\vec{\mathbf{v}}_1$ (Fig. 5–11c); $\Delta\vec{\mathbf{v}}$ will be essentially perpendicular to them. Thus $\Delta\vec{\mathbf{v}}$ points toward the center of the circle. Since $\vec{\mathbf{a}}$, by definition, is in the same direction as $\Delta\vec{\mathbf{v}}$, it too must point toward the center of the circle. Therefore, this acceleration is called **centripetal acceleration** ("center-pointing" acceleration) or **radial acceleration** (since it is directed along the radius, toward the center of the circle), and we denote it by $\vec{\mathbf{a}}_R$.

We next determine the magnitude of the radial (centripetal) acceleration, a_R. Because CA in Fig. 5–11a is perpendicular to $\vec{\mathbf{v}}_1$, and CB is perpendicular to $\vec{\mathbf{v}}_2$, it follows that the angle $\Delta\theta$, defined as the angle between CA and CB, is also the angle between $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$. Hence the vectors $\vec{\mathbf{v}}_1$, $\vec{\mathbf{v}}_2$, and $\Delta\vec{\mathbf{v}}$ in Fig. 5–11b form a triangle that is geometrically similar† to triangle CAB in Fig. 5–11a. If we take $\Delta\theta$ to be very small (letting Δt be very small) and setting $v = v_1 = v_2$ because the magnitude of the velocity is assumed not to change, we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta\ell}{r},$$

or

$$\Delta v \approx \frac{v}{r}\Delta\ell.$$

This is an exact equality when Δt approaches zero, for then the arc length $\Delta\ell$ equals the chord length AB. We want to find the instantaneous acceleration, a_R, so we use the expression above to write

$$a_R = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{r}\frac{\Delta\ell}{\Delta t}.$$

Then, because

$$\lim_{\Delta t \to 0} \frac{\Delta\ell}{\Delta t}$$

is just the linear speed, v, of the object, we have for the centripetal (radial) acceleration

$$a_R = \frac{v^2}{r}. \qquad \text{[centripetal (radial) acceleration]} \quad \textbf{(5–1)}$$

Equation 5–1 is valid even when v is not constant.

To summarize, *an object moving in a circle of radius r at constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is* $a_R = v^2/r$. It is not surprising that this acceleration depends on v and r. The greater the speed v, the faster the velocity changes direction; and the larger the radius, the less rapidly the velocity changes direction.

The acceleration vector points toward the center of the circle. But the velocity vector always points in the direction of motion, which is tangential to the circle. Thus the velocity and acceleration vectors are perpendicular to each other at every point in the path for uniform circular motion (Fig. 5–12). This is another example that illustrates the error in thinking that acceleration and velocity are always in the same direction. For an object falling vertically, $\vec{\mathbf{a}}$ and $\vec{\mathbf{v}}$ are indeed parallel. But in circular motion, $\vec{\mathbf{a}}$ and $\vec{\mathbf{v}}$ are perpendicular, not parallel (nor were they parallel in projectile motion, Section 3–7).

EXERCISE C Can Equations 2–12, the kinematic equations for constant acceleration, be used for uniform circular motion? For example, could Eq. 2–12b be used to calculate the time for the revolving ball in Fig. 5–12 to make one revolution?

†Appendix A contains a review of geometry.

Circular motion is often described in terms of the **frequency** f, the number of revolutions per second. The **period** T of an object revolving in a circle is the time required for one complete revolution. Period and frequency are related by

$$T = \frac{1}{f}. \quad \textbf{(5–2)}$$

For example, if an object revolves at a frequency of 3 rev/s, then each revolution takes $\frac{1}{3}$s. For an object revolving in a circle (of circumference $2\pi r$) at constant speed v, we can write

$$v = \frac{2\pi r}{T},$$

since in one revolution the object travels one circumference.

EXAMPLE 5–8 Acceleration of a revolving ball. A 150-g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m, as in Fig. 5–10 or 5–12. The ball makes 2.00 revolutions in a second. What is its centripetal acceleration?

APPROACH The centripetal acceleration is $a_R = v^2/r$. We are given r, and we can find the speed of the ball, v, from the given radius and frequency.

SOLUTION If the ball makes two complete revolutions per second, then the ball travels in a complete circle in a time interval equal to 0.500 s, which is its period T. The distance traveled in this time is the circumference of the circle, $2\pi r$, where r is the radius of the circle. Therefore, the ball has speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.600\,\mathrm{m})}{(0.500\,\mathrm{s})} = 7.54\,\mathrm{m/s}.$$

The centripetal acceleration† is

$$a_R = \frac{v^2}{r} = \frac{(7.54\,\mathrm{m/s})^2}{(0.600\,\mathrm{m})} = 94.7\,\mathrm{m/s^2}.$$

EXERCISE D If the radius is doubled to 1.20 m but the period stays the same, by what factor will the centripetal acceleration change? (a) 2, (b) 4, (c) $\frac{1}{2}$, (d) $\frac{1}{4}$, (e) none of these.

EXAMPLE 5–9 Moon's centripetal acceleration. The Moon's nearly circular orbit about the Earth has a radius of about 384,000 km and a period T of 27.3 days. Determine the acceleration of the Moon toward the Earth.

APPROACH Again we need to find the velocity v in order to find a_R. We will need to convert to SI units to get v in m/s.

SOLUTION In one orbit around the Earth, the Moon travels a distance $2\pi r$, where $r = 3.84 \times 10^8\,\mathrm{m}$ is the radius of its circular path. The time required for one complete orbit is the Moon's period of 27.3 d. The speed of the Moon in its orbit about the Earth is $v = 2\pi r/T$. The period T in seconds is $T = (27.3\,\mathrm{d})(24.0\,\mathrm{h/d})(3600\,\mathrm{s/h}) = 2.36 \times 10^6\,\mathrm{s}$. Therefore,

$$a_R = \frac{v^2}{r} = \frac{(2\pi r)^2}{T^2 r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(3.84 \times 10^8\,\mathrm{m})}{(2.36 \times 10^6\,\mathrm{s})^2}$$
$$= 0.00272\,\mathrm{m/s^2} = 2.72 \times 10^{-3}\,\mathrm{m/s^2}.$$

We can write this acceleration in terms of $g = 9.80\,\mathrm{m/s^2}$ (the acceleration of gravity at the Earth's surface) as

$$a = 2.72 \times 10^{-3}\,\mathrm{m/s^2}\left(\frac{g}{9.80\,\mathrm{m/s^2}}\right) = 2.78 \times 10^{-4} g.$$

NOTE The centripetal acceleration of the Moon, $a = 2.78 \times 10^{-4}\,g$, is *not* the acceleration of gravity for objects at the Moon's surface due to the Moon's gravity. Rather, it is the acceleration due to the *Earth's* gravity for any object (such as the Moon) that is 384,000 km from the Earth. Notice how small this acceleration is compared to the acceleration of objects near the Earth's surface.

CAUTION

Distinguish the Moon's gravity on objects at its surface, from the Earth's gravity acting on the Moon (this Example)

†Differences in the final digit can depend on whether you keep all digits in your calculator for v (which gives $a_R = 94.7\,\mathrm{m/s^2}$), or if you use $v = 7.54\,\mathrm{m/s}$ in which case you get $a_R = 94.8\,\mathrm{m/s^2}$. Both results are valid since our assumed accuracy is about $\pm 0.1\,\mathrm{m/s}$ (see Section 1–3).

PHYSICS APPLIED
Centrifuge

*Centrifugation

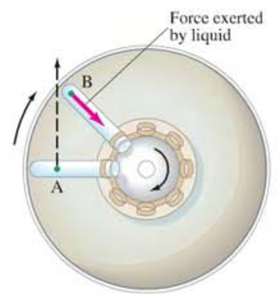


FIGURE 5–13 Two positions of a rotating test tube in a centrifuge (top view). At A, the green dot represents a macromolecule or other particle being sedimented. It would tend to follow the dashed line, heading toward the bottom of the tube, but the fluid resists this motion by exerting a force on the particle as shown at point B.

Centrifuges and very high speed ultracentrifuges, are used to sediment materials quickly or to separate materials. Test tubes held in the centrifuge rotor are accelerated to very high rotational speeds: see Fig. 5–13, where one test tube is shown in two positions as the rotor turns. The small green dot represents a small particle, perhaps a macromolecule, in a fluid-filled test tube. At position A the particle has a tendency to move in a straight line, but the fluid resists the motion of the particles, exerting a centripetal force that keeps the particles moving nearly in a circle. The resistive force exerted by the fluid (liquid, gas, or gel, depending on the application) usually does not quite equal mv^2/r, and the particles move slowly toward the bottom of the tube. A centrifuge provides an "effective gravity" much larger than normal gravity because of the high rotational speeds, thus causing more rapid sedimentation.

EXAMPLE 5–10 Ultracentrifuge. The rotor of an ultracentrifuge rotates at 50,000 rpm (revolutions per minute). A particle at the top of a test tube (Fig. 5–13) is 6.00 cm from the rotation axis. Calculate its centripetal acceleration, in "g's."

APPROACH We calculate the centripetal acceleration from $a_R = v^2/r$.

SOLUTION The test tube makes 5.00×10^4 revolutions each minute, or, dividing by 60 s/min, 833 rev/s. The time to make one revolution, the period T, is

$$T = \frac{1}{(833\ \text{rev/s})} = 1.20 \times 10^{-3}\ \text{s/rev}.$$

At the top of the tube, a particle revolves in a circle of circumference $2\pi r = (2\pi)(0.0600\ \text{m}) = 0.377$ m per revolution. The speed of the particle is then

$$v = \frac{2\pi r}{T} = \left(\frac{0.377\ \text{m/rev}}{1.20 \times 10^{-3}\ \text{s/rev}}\right) = 3.14 \times 10^2\ \text{m/s}.$$

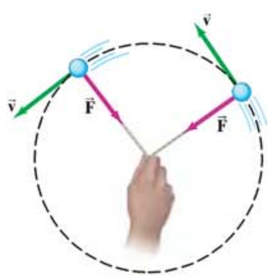
The centripetal acceleration is

$$a_R = \frac{v^2}{r} = \frac{(3.14 \times 10^2\ \text{m/s})^2}{0.0600\ \text{m}} = 1.64 \times 10^6\ \text{m/s}^2,$$

which, dividing by $g = 9.80\ \text{m/s}^2$, is 1.67×10^5 g's = 167,000 g's.

5–3 Dynamics of Uniform Circular Motion

FIGURE 5–14 A force is required to keep an object moving in a circle. If the speed is constant, the force is directed toward the circle's center.



According to Newton's second law $(\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}})$, an object that is accelerating must have a net force acting on it. An object moving in a circle, such as a ball on the end of a string, must therefore have a force applied to it to keep it moving in that circle. That is, a net force is necessary to give it centripetal acceleration. The magnitude of the required force can be calculated using Newton's second law for the radial component, $\Sigma F_R = ma_R$, where a_R is the centripetal acceleration, $a_R = v^2/r$, and ΣF_R is the total (or net) force in the radial direction:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}. \qquad \text{[circular motion]} \quad \textbf{(5–3)}$$

For uniform circular motion (v = constant), the acceleration is a_R, which is directed toward the center of the circle at any moment. Thus the *net force too must be directed toward the center of the circle*, Fig. 5–14. A net force is necessary because if no net force were exerted on the object, it would not move in a circle but in a straight line, as Newton's first law tells us. The direction of the net force is continually changing so that it is always directed toward the center of the circle. This force is sometimes called a centripetal ("pointing toward the center") force. But be aware that "centripetal force" does not indicate some new kind of force. The term merely describes the *direction* of the net force needed to provide a circular path: the net force is directed toward the circle's center. The force *must be applied by other objects*. For example, to swing a ball in a circle on the end of a string, you pull on the string and the string exerts the force on the ball. (Try it.)

CAUTION
Centripetal force is not a new kind of force (Every force must be exerted by an object)

There is a common misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal ("center-fleeing") force. This is incorrect: *there is no outward force* on the revolving object. Consider, for example, a person swinging a ball on the end of a string around her head (Fig. 5–15). If you have ever done this yourself, you know that you feel a force pulling outward on your hand. The misconception arises when this pull is interpreted as an outward "centrifugal" force pulling on the ball that is transmitted along the string to your hand. This is not what is happening at all. To keep the ball moving in a circle, you pull *inwardly* on the string, and the string exerts this force on the ball. The ball exerts an equal and opposite force on the string (Newton's third law), and *this* is the outward force your hand feels (see Fig. 5–15).

The force *on the ball* is the one exerted *inwardly* on it by you, via the string. To see even more convincing evidence that a "centrifugal force" does not act on the ball, consider what happens when you let go of the string. If a centrifugal force were acting, the ball would fly outward, as shown in Fig. 5–16a. But it doesn't; the ball flies off tangentially (Fig. 5–16b), in the direction of the velocity it had at the moment it was released, because the inward force no longer acts. Try it and see!

CAUTION

There is no real "centrifugal force"

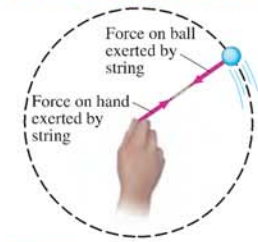
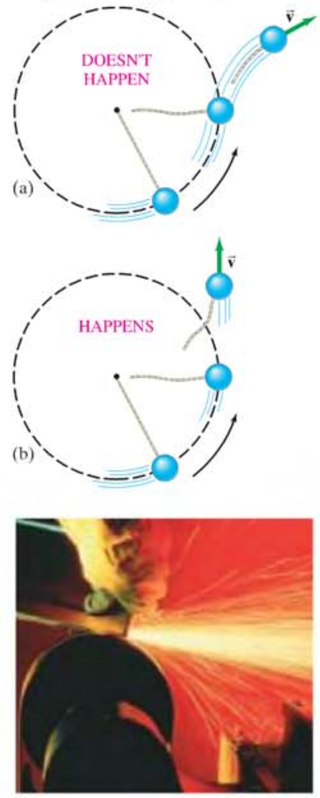


FIGURE 5–15 Swinging a ball on the end of a string.

FIGURE 5–16 If centrifugal force existed, the revolving ball would fly outward as in (a) when released. In fact, it flies off tangentially as in (b). For example, in (c) sparks fly in straight lines tangentially from the edge of a rotating grinding wheel.



EXERCISE E Return to the Chapter-Opening Question, page 112, and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 5–11 ESTIMATE **Force on revolving ball (horizontal).** Estimate the force a person must exert on a string attached to a 0.150-kg ball to make the ball revolve in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions per second ($T = 0.500\,\text{s}$), as in Example 5–8. Ignore the string's mass.

APPROACH First we need to draw the free-body diagram for the ball. The forces acting on the ball are the force of gravity, $m\vec{\mathbf{g}}$ downward, and the tension force $\vec{\mathbf{F}}_T$ that the string exerts toward the hand at the center (which occurs because the person exerts that same force on the string). The free-body diagram for the ball is as shown in Fig. 5–17. The ball's weight complicates matters and makes it impossible to revolve a ball with the cord perfectly horizontal. We assume the weight is small, and put $\phi \approx 0$ in Fig. 5–17. Thus $\vec{\mathbf{F}}_T$ will act nearly horizontally and, in any case, provides the force necessary to give the ball its centripetal acceleration.

SOLUTION We apply Newton's second law to the radial direction, which we assume is horizontal:

$$(\Sigma F)_R = ma_R,$$

where $a_R = v^2/r$ and $v = 2\pi r/T = 2\pi(0.600\,\text{m})/(0.500\,\text{s}) = 7.54\,\text{m/s}$. Thus

$$F_T = m\frac{v^2}{r} = (0.150\,\text{kg})\frac{(7.54\,\text{m/s})^2}{(0.600\,\text{m})} \approx 14\,\text{N}.$$

NOTE We keep only two significant figures in the answer because we ignored the ball's weight; it is $mg = (0.150\,\text{kg})(9.80\,\text{m/s}^2) = 1.5\,\text{N}$, about $\frac{1}{10}$ of our result, which is small but not so small as to justify stating a more precise answer for F_T.

NOTE To include the effect of $m\vec{\mathbf{g}}$, resolve $\vec{\mathbf{F}}_T$ in Fig. 5–17 into components, and set the horizontal component of $\vec{\mathbf{F}}_T$ equal to mv^2/r and its vertical component equal to mg.

FIGURE 5–17 Example 5–11.

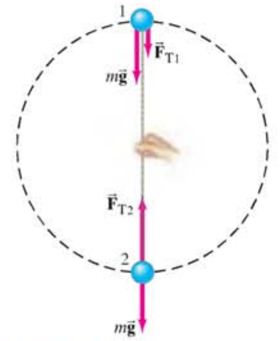


FIGURE 5–18 Example 5–12. Free-body diagrams for positions 1 and 2.

CAUTION
Circular motion only if cord is under tension

EXAMPLE 5–12 Revolving ball (vertical circle). A 0.150-kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a *vertical* circle. (*a*) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (*b*) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (*a*).

APPROACH The ball moves in a vertical circle and is *not* undergoing uniform circular motion. The radius is assumed constant, but the speed v changes because of gravity. Nonetheless, Eq. 5–1 is valid at each point along the circle, and we use it at the top and bottom points. The free-body diagram is shown in Fig. 5–18 for both positions.

SOLUTION (*a*) At the top (point 1), two forces act on the ball: $m\vec{g}$, the force of gravity, and $\vec{F}_{T1}$, the tension force the cord exerts at point 1. Both act downward, and their vector sum acts to give the ball its centripetal acceleration a_R. We apply Newton's second law, for the vertical direction, choosing downward as positive since the acceleration is downward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T1} + mg = m\frac{v_1^2}{r}. \qquad \text{[at top]}$$

From this equation we can see that the tension force F_{T1} at point 1 will get larger if v_1 (ball's speed at top of circle) is made larger, as expected. But we are asked for the *minimum* speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it. But if the tension disappears (because v_1 is too small) the cord can go limp, and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{T1} = 0$, for which we have

$$mg = m\frac{v_1^2}{r}. \qquad \text{[minimum speed at top]}$$

We solve for v_1, keeping an extra digit for use in (*b*):

$$v_1 = \sqrt{gr} = \sqrt{(9.80\,\text{m/s}^2)(1.10\,\text{m})} = 3.283\,\text{m/s}.$$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

(*b*) When the ball is at the bottom of the circle (point 2 in Fig. 5–18), the cord exerts its tension force F_{T2} upward, whereas the force of gravity, $m\vec{g}$, still acts downward. Choosing *upward* as positive, Newton's second law gives:

$$(\Sigma F)_R = ma_R$$

$$F_{T2} - mg = m\frac{v_2^2}{r}. \qquad \text{[at bottom]}$$

The speed v_2 is given as twice that in (*a*), namely 6.566 m/s. We solve for F_{T2}:

$$F_{T2} = m\frac{v_2^2}{r} + mg$$

$$= (0.150\,\text{kg})\frac{(6.566\,\text{m/s})^2}{(1.10\,\text{m})} + (0.150\,\text{kg})(9.80\,\text{m/s}^2) = 7.35\,\text{N}.$$

EXERCISE F A rider on a Ferris wheel moves in a vertical circle of radius r at constant speed v (Fig. 5–19). Is the normal force that the seat exerts on the rider at the top of the wheel (*a*) less than, (*b*) more than, or (*c*) the same as, the force the seat exerts at the bottom of the wheel?

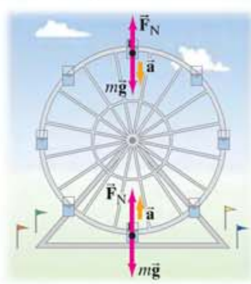


FIGURE 5–19 Exercise F.

EXAMPLE 5–13 **Conical pendulum.** A small ball of mass m, suspended by a cord of length ℓ, revolves in a circle of radius $r = \ell \sin\theta$, where θ is the angle the string makes with the vertical (Fig. 5–20). (*a*) In what direction is the acceleration of the ball, and what causes the acceleration? (*b*) Calculate the speed and period (time required for one revolution) of the ball in terms of ℓ, θ, g, and m.

APPROACH We can answer (*a*) by looking at Fig. 5–20, which shows the forces on the revolving ball at one instant: the acceleration points horizontally toward the center of the ball's circular path (not along the cord). The force responsible for the acceleration is the *net* force which here is the vector sum of the forces acting on the mass m: its weight $\vec{\mathbf{F}}_G$ (of magnitude $F_G = mg$) and the force exerted by the tension in the cord, $\vec{\mathbf{F}}_T$. The latter has horizontal and vertical components of magnitude $F_T \sin\theta$ and $F_T \cos\theta$, respectively.

SOLUTION (*b*) We apply Newton's second law to the horizontal and vertical directions. In the vertical direction, there is no motion, so the acceleration is zero and the net force in the vertical direction is zero:

$$F_T \cos\theta - mg = 0.$$

In the horizontal direction there is only one force, of magnitude $F_T \sin\theta$, that acts toward the center of the circle and gives rise to the acceleration v^2/r. Newton's second law tells us:

$$F_T \sin\theta = m\frac{v^2}{r}.$$

We solve the second equation for v, and substitute for F_T from the first equation (and use $r = \ell \sin\theta$):

$$v = \sqrt{\frac{rF_T \sin\theta}{m}} = \sqrt{\frac{r}{m}\left(\frac{mg}{\cos\theta}\right)\sin\theta} = \sqrt{\frac{\ell g \sin^2\theta}{\cos\theta}}.$$

The period T is the time required to make one revolution, a distance of $2\pi r = 2\pi\ell \sin\theta$. The speed v can thus be written $v = 2\pi\ell\sin\theta/T$; then

$$T = \frac{2\pi\ell\sin\theta}{v} = \frac{2\pi\ell\sin\theta}{\sqrt{\dfrac{\ell g \sin^2\theta}{\cos\theta}}} = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}.$$

NOTE The speed and period do not depend on the mass m of the ball. They do depend on ℓ and θ.

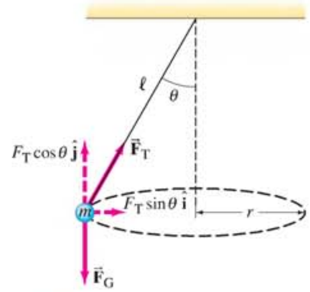


FIGURE 5–20 Example 5–13. Conical pendulum.

PROBLEM SOLVING

Uniform Circular Motion

1. **Draw a free-body diagram**, showing all the forces acting on each object under consideration. Be sure you can identify the source of each force (tension in a cord, Earth's gravity, friction, normal force, and so on). Don't put in something that doesn't belong (like a centrifugal force).
2. **Determine** which of the forces, or which of their components, act to provide the centripetal acceleration—that is, all the **forces or components that act radially**, toward or away from the center of the circular path. The sum of these forces (or components) provides the centripetal acceleration, $a_R = v^2/r$.
3. **Choose a convenient coordinate system**, preferably with one axis along the acceleration direction.
4. **Apply Newton's second law** to the radial component:

$$(\Sigma F)_R = ma_R = m\frac{v^2}{r}. \quad \text{[radial direction]}$$

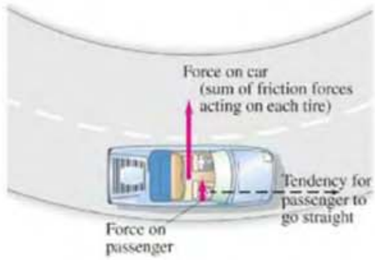


FIGURE 5–21 The road exerts an inward force on a car (friction against the tires) to make it move in a circle. The car exerts an inward force on the passenger.

FIGURE 5–22 Race car heading into a curve. From the tire marks we see that most cars experienced a sufficient friction force to give them the needed centripetal acceleration for rounding the curve safely. But, we also see tire tracks of cars on which there was not sufficient force—and which unfortunately followed more nearly straight-line paths.

5–4 Highway Curves: Banked and Unbanked

An example of circular dynamics occurs when an automobile rounds a curve, say to the left. In such a situation, you may feel that you are thrust outward toward the right side door. But there is no mysterious centrifugal force pulling on you. What is happening is that you tend to move in a straight line, whereas the car has begun to follow a curved path. To make you go in the curved path, the seat (friction) or the door of the car (direct contact) exerts a force on you (Fig. 5–21). The car also must have a force exerted on it toward the center of the curve if it is to move in that curve. On a flat road, this force is supplied by friction between the tires and the pavement.

If the wheels and tires of the car are rolling normally without slipping or sliding, the bottom of the tire is at rest against the road at each instant; so the friction force the road exerts on the tires is static friction. But if the static friction force is not great enough, as under icy conditions or high speed, sufficient friction force cannot be applied and the car will skid out of a circular path into a more nearly straight path. See Fig. 5–22. Once a car skids or slides, the friction force becomes kinetic friction, which is less than static friction.

EXAMPLE 5–14 **Skidding on a curve.** A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 15 m/s (54 km/h). Will the car follow the curve, or will it skid? Assume: (*a*) the pavement is dry and the coefficient of static friction is $\mu_s = 0.60$; (*b*) the pavement is icy and $\mu_s = 0.25$.

APPROACH The forces on the car are gravity mg downward, the normal force F_N exerted upward by the road, and a horizontal friction force due to the road. They are shown in Fig. 5–23, which is the free-body diagram for the car. The car will follow the curve if the maximum static friction force is greater than the mass times the centripetal acceleration.

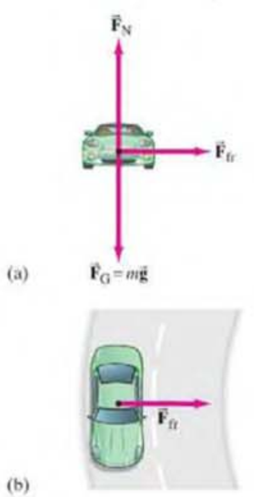


FIGURE 5–23 Example 5–14. Forces on a car rounding a curve on a flat road. (a) Front view, (b) top view.

SOLUTION In the vertical direction there is no acceleration. Newton's second law tells us that the normal force F_N on the car is equal to the weight mg:

$$F_N = mg = (1000\,\text{kg})(9.80\,\text{m/s}^2) = 9800\,\text{N}.$$

In the horizontal direction the only force is friction, and we must compare it to the force needed to produce the centripetal acceleration to see if it is sufficient. The net horizontal force required to keep the car moving in a circle around the curve is

$$(\Sigma F)_R = ma_R = m\frac{v^2}{r} = (1000\,\text{kg})\frac{(15\,\text{m/s})^2}{(50\,\text{m})} = 4500\,\text{N}.$$

Now we compute the maximum total static friction force (the sum of the friction forces acting on each of the four tires) to see if it can be large enough to provide a safe centripetal acceleration. For (*a*), $\mu_s = 0.60$, and the maximum friction force attainable (recall from Section 5–1 that $F_{fr} \leq \mu_s F_N$) is

$$(F_{fr})_{max} = \mu_s F_N = (0.60)(9800\,\text{N}) = 5880\,\text{N}.$$

Since a force of only 4500 N is needed, and that is, in fact, how much will be exerted by the road as a static friction force, the car can follow the curve. But in (*b*) the maximum static friction force possible is

$$(F_{\rm fr})_{\rm max} = \mu_s F_N = (0.25)(9800\,{\rm N}) = 2450\,{\rm N}.$$

The car will skid because the ground cannot exert sufficient force (4500 N is needed) to keep it moving in a curve of radius 50 m at a speed of 54 km/h.

PHYSICS APPLIED
Banked curves

The banking of curves can reduce the chance of skidding. The normal force exerted by a banked road, acting perpendicular to the road, will have a component toward the center of the circle (Fig. 5–24), thus reducing the reliance on friction. For a given banking angle θ, there will be one speed for which no friction at all is required. This will be the case when the horizontal component of the normal force toward the center of the curve, $F_N \sin\theta$ (see Fig. 5–24), is just equal to the force required to give a vehicle its centripetal acceleration—that is, when

$$F_N \sin\theta = m\frac{v^2}{r}. \qquad \text{[no friction required]}$$

The banking angle of a road, θ, is chosen so that this condition holds for a particular speed, called the "design speed."

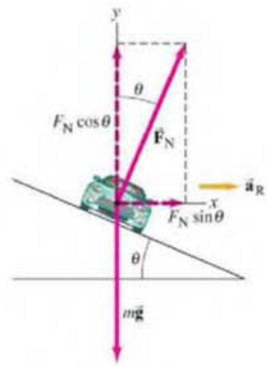


FIGURE 5–24 Normal force on a car rounding a banked curve, resolved into its horizontal and vertical components. The centripetal acceleration is horizontal (*not* parallel to the sloping road). The friction force on the tires, not shown, could point up or down along the slope, depending on the car's speed. The friction force will be zero for one particular speed.

EXAMPLE 5–15 **Banking angle.** (*a*) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required. (*b*) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

APPROACH Even though the road is banked, the car is still moving along a horizontal circle, so the centripetal acceleration needs to be horizontal. We choose our x and y axes as horizontal and vertical so that a_R, which is horizontal, is along the x axis. The forces on the car are the Earth's gravity mg downward, and the normal force F_N exerted by the road perpendicular to its surface. See Fig. 5–24, where the components of F_N are also shown. We don't need to consider the friction of the road because we are designing a road to be banked so as to eliminate dependence on friction.

SOLUTION (*a*) Since there is no vertical motion, $\Sigma F_y = ma_y$ gives us

$$F_N \cos\theta - mg = 0.$$

Thus,

$$F_N = \frac{mg}{\cos\theta}.$$

[Note in this case that $F_N \geq mg$ since $\cos\theta \leq 1$.]

CAUTION
F_N is not always equal to mg

We substitute this relation for F_N into the equation for the horizontal motion,

$$F_N \sin\theta = m\frac{v^2}{r},$$

and obtain

$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

or

$$\tan\theta = \frac{v^2}{rg}.$$

This is the formula for the banking angle θ: no friction needed at speed v.

(*b*) For $r = 50$ m and $v = 50$ km/h (or 14 m/s),

$$\tan\theta = \frac{(14\,{\rm m/s})^2}{(50\,{\rm m})(9.8\,{\rm m/s^2})} = 0.40,$$

so $\theta = 22°$.

EXERCISE G The banking angle of a curve for a design speed v is θ_1. What banking angle θ_2 is needed for a design speed of $2v$? (*a*) $\theta_2 = 4\theta_1$; (*b*) $\theta_2 = 2\theta_1$; (*c*) $\tan\theta_2 = 4\tan\theta_1$; (*d*) $\tan\theta_2 = 2\tan\theta_1$.

EXERCISE H Can a heavy truck and a small car travel safely at the same speed around an icy banked-curve road?

*5–5 Nonuniform Circular Motion

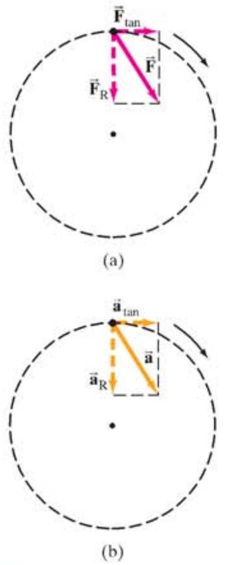


FIGURE 5–25 The speed of an object moving in a circle changes if the force on it has a tangential component, F_{tan}. Part (a) shows the force $\vec{F}$ and its vector components; part (b) shows the acceleration vector and its vector components.

Circular motion at constant speed occurs when the net force on an object is exerted toward the center of the circle. If the net force is not directed toward the center but is at an angle, as shown in Fig. 5–25a, the force has two components. The component directed toward the center of the circle, $\vec{F}_R$, gives rise to the centripetal acceleration, $\vec{a}_R$, and keeps the object moving in a circle. The component tangent to the circle, $\vec{F}_{tan}$, acts to increase (or decrease) the speed, and thus gives rise to a component of the acceleration tangent to the circle, $\vec{a}_{tan}$. When the speed of the object is changing, a tangential component of force is acting.

When you first start revolving a ball on the end of a string around your head, you must give it tangential acceleration. You do this by pulling on the string with your hand displaced from the center of the circle. In athletics, a hammer thrower accelerates the hammer tangentially in a similar way so that it reaches a high speed before release.

The tangential component of the acceleration, a_{tan}, has magnitude equal to the rate of change of the *magnitude* of the object's velocity:

$$a_{tan} = \frac{dv}{dt}. \qquad \textbf{(5–4)}$$

The radial (centripetal) acceleration arises from the change in *direction* of the velocity and, as we have seen, has magnitude

$$a_R = \frac{v^2}{r}.$$

The tangential acceleration always points in a direction tangent to the circle, and is in the direction of motion (parallel to $\vec{v}$, which is always tangent to the circle) if the speed is increasing, as shown in Fig. 5–25b. If the speed is decreasing, $\vec{a}_{tan}$ points antiparallel to $\vec{v}$. In either case, $\vec{a}_{tan}$ and $\vec{a}_R$ are always perpendicular to each other; and *their directions change* continually as the object moves along its circular path. The total vector acceleration $\vec{a}$ is the sum of the two components:

$$\vec{a} = \vec{a}_{tan} + \vec{a}_R. \qquad \textbf{(5–5)}$$

Since $\vec{a}_R$ and $\vec{a}_{tan}$ are always perpendicular to each other, the magnitude of $\vec{a}$ at any moment is

$$a = \sqrt{a_{tan}^2 + a_R^2}.$$

EXAMPLE 5–16 **Two components of acceleration.** A race car starts from rest in the pit area and accelerates at a uniform rate to a speed of 35 m/s in 11 s, moving on a circular track of radius 500 m. Assuming constant tangential acceleration, find (*a*) the tangential acceleration, and (*b*) the radial acceleration, at the instant when the speed is $v = 15\,\mathrm{m/s}$.

APPROACH The tangential acceleration relates to the change in speed of the car, and can be calculated as $a_{tan} = \Delta v/\Delta t$. The centripetal acceleration relates to the change in the *direction* of the velocity vector and is calculated using $a_R = v^2/r$.

SOLUTION (*a*) During the 11-s time interval, we assume the tangential acceleration a_{tan} is constant. Its magnitude is

$$a_{tan} = \frac{\Delta v}{\Delta t} = \frac{(35\,\mathrm{m/s} - 0\,\mathrm{m/s})}{11\,\mathrm{s}} = 3.2\,\mathrm{m/s^2}.$$

(*b*) When $v = 15\,\mathrm{m/s}$, the centripetal acceleration is

$$a_R = \frac{v^2}{r} = \frac{(15\,\mathrm{m/s})^2}{(500\,\mathrm{m})} = 0.45\,\mathrm{m/s^2}.$$

NOTE The radial acceleration increases continually, whereas the tangential acceleration stays constant.

EXERCISE I When the speed of the race car in Example 5–16 is 30 m/s, how are (*a*) a_{tan} and (*b*) a_R changed?

These concepts can be used for an object moving along any curved path, such as that shown in Fig. 5–26. We can treat any portion of the curve as an arc of a circle with a "radius of curvature" r. The velocity at any point is always tangent to the path. The acceleration can be written, in general, as a vector sum of two components: the tangential component $a_{\tan} = dv/dt$, and the radial (centripetal) component $a_R = v^2/r$.

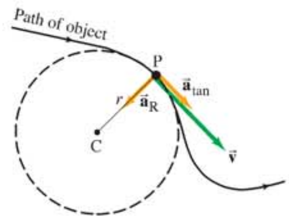


FIGURE 5–26 Object following a curved path (solid line). At point P the path has a radius of curvature r. The object has velocity $\vec{v}$, tangential acceleration $\vec{a}_{\tan}$ (the object is here increasing in speed), and radial (centripetal) acceleration $\vec{a}_R$ (magnitude $a_R = v^2/r$) which points toward the center of curvature C.

*5–6 Velocity-Dependent Forces: Drag and Terminal Velocity

When an object slides along a surface, the force of friction acting on the object is nearly independent of how fast the object is moving. But other types of resistive forces do depend on the object's velocity. The most important example is for an object moving through a liquid or gas, such as air. The fluid offers resistance to the motion of the object, and this resistive force, or **drag force**, depends on the velocity of the object.[†]

The way the drag force varies with velocity is complicated in general. But for small objects at very low speeds, a good approximation can often be made by assuming that the drag force, F_D, is directly proportional to the magnitude of the velocity, v:

$$F_D = -bv. \tag{5–6}$$

The minus sign is necessary because the drag force opposes the motion. Here b is a constant (approximately) that depends on the viscosity of the fluid and on the size and shape of the object. Equation 5–6 works well for small objects moving at low speed in a viscous liquid. It also works for very small objects moving in air at very low speeds, such as dust particles. For objects moving at high speeds, such as an airplane, a sky diver, a baseball, or an automobile, the force of air resistance can be better approximated as being proportional to v^2:

$$F_D \propto v^2.$$

For accurate calculations, however, more complicated forms and numerical integration generally need to be used. For objects moving through liquids, Eq. 5–6 works well for everyday objects at normal speeds (e.g., a boat in water).

Let us consider an object that falls from rest, through air or other fluid, under the action of gravity and a resistive force proportional to v. The forces acting on the object are the force of gravity, mg, acting downward, and the drag force, $-bv$, acting upward (Fig. 5–27a). Since the velocity $\vec{v}$ points downward, let us take the positive direction as downward. Then the net force on the object can be written

$$\Sigma F = mg - bv.$$

From Newton's second law $\Sigma F = ma$, we have

$$mg - bv = m\frac{dv}{dt}, \tag{5–7}$$

where we have written the acceleration according to its definition as rate of change of velocity, $a = dv/dt$. At $t = 0$, we set $v = 0$ and the acceleration $dv/dt = g$. As the object falls and increases in speed, the resistive force increases, and this reduces the acceleration, dv/dt (see Fig. 5–27b). The velocity continues to increase, but at a slower rate. Eventually, the velocity becomes so large that the magnitude of the resistive force, bv, approaches that of the gravitational force, mg; when the two are equal, we have

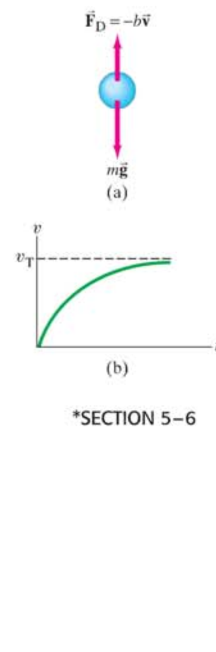
$$mg - bv = 0. \tag{5–8}$$

At this point $dv/dt = 0$ and the object no longer increases in speed. It has reached its **terminal velocity** and continues to fall at this constant velocity until it hits the ground. This sequence of events is shown in the graph of Fig. 5–27b. The value of the terminal velocity v_T can be obtained from Eq. 5–8.

$$v_T = \frac{mg}{b}. \tag{5–9}$$

If the resistive force is assumed proportional to v^2, or an even higher power of v, the sequence of events is similar and a terminal velocity reached, although it will not be given by Eq. 5–9.

FIGURE 5–27 (a) Forces acting on an object falling downward. (b) Graph of the velocity of an object falling due to gravity when the air resistance drag force is $F_D = -bv$. Initially, $v = 0$ and $dv/dt = g$, but as time goes on dv/dt (= slope of curve) decreases because of F_D. Eventually, v approaches a maximum value, v_T, the terminal velocity, which occurs when F_D has magnitude equal to mg.



[†]Any buoyant force (Chapter 13) is ignored in this Section.

EXAMPLE 5–17 **Force proportional to velocity.** Determine the velocity as a function of time for an object falling vertically from rest when there is a resistive force linearly proportional to v.

APPROACH This is a derivation and we start with Eq. 5–7, which we rewrite as

$$\frac{dv}{dt} = g - \frac{b}{m}v.$$

SOLUTION In this equation there are two variables, v and t. We collect variables of the same type on one or the other side of the equation:

$$\frac{dv}{g - \frac{b}{m}v} = dt \qquad \text{or} \qquad \frac{dv}{v - \frac{mg}{b}} = -\frac{b}{m}dt.$$

Now we can integrate, remembering $v = 0$ at $t = 0$:

$$\int_0^v \frac{dv}{v - \frac{mg}{b}} = -\frac{b}{m}\int_0^t dt$$

which gives

$$\ln\left(v - \frac{mg}{b}\right) - \ln\left(-\frac{mg}{b}\right) = -\frac{b}{m}t$$

or

$$\ln\frac{v - mg/b}{-mg/b} = -\frac{b}{m}t.$$

We raise each side to the exponential [note that the natural log and the exponential are inverse operations of each other: $e^{\ln x} = x$, or $\ln(e^x) = x$] and obtain

$$v - \frac{mg}{b} = -\frac{mg}{b}e^{-\frac{b}{m}t}$$

so

$$v = \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right).$$

This relation gives the velocity v as a function of time and corresponds to the graph of Fig. 5–27b. As a check, note that at $t = 0$, and $v = 0$

$$a(t=0) = \frac{dv}{dt} = \frac{mg}{b}\frac{d}{dt}\left(1 - e^{-\frac{b}{m}t}\right) = \frac{mg}{b}\left(\frac{b}{m}\right) = g,$$

as expected (see also Eq. 5–7). At large t, $e^{-\frac{b}{m}t}$ approaches zero, so v approaches mg/b, which is the terminal velocity, v_T, as we saw earlier. If we set $\tau = m/b$, then $v = v_T(1 - e^{-t/\tau})$. So $\tau = m/b$ is the time required for the velocity to reach 63% of the terminal velocity (since $e^{-1} = 0.37$). Figure 5–27b shows a plot of speed v vs. time t, where the terminal velocity $v_T = mg/b$.

Summary

When two objects slide over one another, the force of **friction** that each exerts on the other can be written approximately as $F_{fr} = \mu_k F_N$, where F_N is the **normal force** (the force each object exerts on the other perpendicular to their contact surfaces), and μ_k is the coefficient of **kinetic friction**. If the objects are at rest relative to each other, even though forces act, then F_{fr} is just large enough to hold them at rest and satisfies the inequality $F_{fr} \le \mu_s F_N$, where μ_s is the coefficient of **static friction**.

An object moving in a circle of radius r with constant speed v is said to be in **uniform circular motion**. It has a **radial acceleration** a_R that is directed radially toward the center of the circle (also called **centripetal acceleration**), and has magnitude

$$a_R = \frac{v^2}{r}. \qquad (5\text{–}1)$$

The direction of the velocity vector and that of the acceleration $\vec{\mathbf{a}}_R$ are continually changing in direction, but are perpendicular to each other at each moment.

A force is needed to keep an object revolving uniformly in a circle, and the direction of this force is toward the center of the circle. This force may be gravity (as for the Moon), or tension in a cord, or a component of the normal force, or another type of force or a combination of forces.

[*When the speed of circular motion is not constant, the acceleration has two components, tangential as well as radial. The force too has tangential and radial components.]

[*A **drag force** acts on an object moving through a fluid, such as air or water. The drag force F_D can often be approximated by $F_D = -bv$ or $F_D \propto v^2$, where v is the speed of the object relative to the fluid.]

Questions

1. A heavy crate rests on the bed of a flatbed truck. When the truck accelerates, the crate remains where it is on the truck, so it, too, accelerates. What force causes the crate to accelerate?
2. A block is given a push so that it slides up a ramp. After the block reaches its highest point, it slides back down, but the magnitude of its acceleration is less on the descent than on the ascent. Why?
3. Why is the stopping distance of a truck much shorter than for a train going the same speed?
4. Can a coefficient of friction exceed 1.0?
5. Cross-country skiers prefer their skis to have a large coefficient of static friction but a small coefficient of kinetic friction. Explain why. [*Hint*: Think of uphill and downhill.]
6. When you must brake your car very quickly, why is it safer if the wheels don't lock? When driving on slick roads, why is it advisable to apply the brakes slowly?
7. When attempting to stop a car quickly on dry pavement, which of the following methods will stop the car in the least time? (*a*) Slam on the brakes as hard as possible, locking the wheels and *skidding* to a stop. (*b*) Press the brakes as hard as possible without locking the wheels and *rolling* to a stop. Explain.
8. You are trying to push your stalled car. Although you apply a horizontal force of 400 N to the car, it doesn't budge, and neither do you. Which force(s) must also have a magnitude of 400 N: (*a*) the force exerted by the car on you; (*b*) the friction force exerted by the car on the road; (*c*) the normal force exerted by the road on you; (*d*) the friction force exerted by the road on you?
9. It is not easy to walk on an icy sidewalk without slipping. Even your gait looks different than on dry pavement. Describe what you need to do differently on the icy surface and why.
10. A car rounds a curve at a steady 50 km/h. If it rounds the same curve at a steady 70 km/h, will its acceleration be any different? Explain.
11. Will the acceleration of a car be the same when a car travels around a sharp curve at a constant 60 km/h as when it travels around a gentle curve at the same speed? Explain.
12. Describe all the forces acting on a child riding a horse on a merry-go-round. Which of these forces provides the centripetal acceleration of the child?
13. A child on a sled comes flying over the crest of a small hill, as shown in Fig. 5–28. His sled does not leave the ground, but he feels the normal force between his chest and the sled decrease as he goes over the hill. Explain this decrease using Newton's second law.

FIGURE 5–28 Question 13.

14. Sometimes it is said that water is removed from clothes in a spin dryer by centrifugal force throwing the water outward. Is this correct? Discuss.
15. Technical reports often specify only the rpm for centrifuge experiments. Why is this inadequate?
16. A girl is whirling a ball on a string around her head in a horizontal plane. She wants to let go at precisely the right time so that the ball will hit a target on the other side of the yard. When should she let go of the string?
17. The game of tetherball is played with a ball tied to a pole with a string. When the ball is struck, it whirls around the pole as shown in Fig. 5–29. In what direction is the acceleration of the ball, and what causes the acceleration?

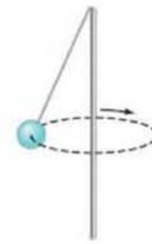


FIGURE 5–29 Problem 17.

18. Astronauts who spend long periods in outer space could be adversely affected by weightlessness. One way to simulate gravity is to shape the spaceship like a cylindrical shell that rotates, with the astronauts walking on the inside surface (Fig. 5–30). Explain how this simulates gravity. Consider (*a*) how objects fall, (*b*) the force we feel on our feet, and (*c*) any other aspects of gravity you can think of.

FIGURE 5–30 Question 18.

19. A bucket of water can be whirled in a vertical circle without the water spilling out, even at the top of the circle when the bucket is upside down. Explain.
20. A car maintains a constant speed v as it traverses the hill and valley shown in Fig. 5–31. Both the hill and valley have a radius of curvature R. At which point, A, B, or C, is the normal force acting on the car (*a*) the largest, (*b*) the smallest? Explain. (*c*) Where would the driver feel heaviest and (*d*) lightest? Explain. (*e*) How fast can the car go without losing contact with the road at A?

FIGURE 5–31 Question 20.

21. Why do bicycle riders lean in when rounding a curve at high speed?
22. Why do airplanes bank when they turn? How would you compute the banking angle given the airspeed and radius of the turn? [*Hint*: Assume an aerodynamic "lift" force acts perpendicular to the wings.]

*23. For a drag force of the form $F = -bv$, what are the units of b?

*24. Suppose two forces act on an object, one force proportional to v and the other proportional to v^2. Which force dominates at high speed?

Problems

5–1 Friction and Newton's Laws

1. (I) If the coefficient of kinetic friction between a 22-kg crate and the floor is 0.30, what horizontal force is required to move the crate at a steady speed across the floor? What horizontal force is required if μ_k is zero?

2. (I) A force of 35.0 N is required to start a 6.0-kg box moving across a horizontal concrete floor. (*a*) What is the coefficient of static friction between the box and the floor? (*b*) If the 35.0-N force continues, the box accelerates at $0.60\ \text{m/s}^2$. What is the coefficient of kinetic friction?

3. (I) Suppose you are standing on a train accelerating at 0.20 *g*. What minimum coefficient of static friction must exist between your feet and the floor if you are not to slide?

4. (I) The coefficient of static friction between hard rubber and normal street pavement is about 0.90. On how steep a hill (maximum angle) can you leave a car parked?

5. (I) What is the maximum acceleration a car can undergo if the coefficient of static friction between the tires and the ground is 0.90?

6. (II) (*a*) A box sits at rest on a rough 33° inclined plane. Draw the free-body diagram, showing all the forces acting on the box. (*b*) How would the diagram change if the box were sliding down the plane. (*c*) How would it change if the box were sliding up the plane after an initial shove?

7. (II) A 25.0-kg box is released on a 27° incline and accelerates down the incline at $0.30\ \text{m/s}^2$. Find the friction force impeding its motion. What is the coefficient of kinetic friction?

8. (II) A car can decelerate at $-3.80\ \text{m/s}^2$ without skidding when coming to rest on a level road. What would its deceleration be if the road is inclined at 9.3° and the car moves uphill? Assume the same static friction coefficient.

9. (II) A skier moves down a 27° slope at constant speed. What can you say about the coefficient of friction, μ_k? Assume the speed is low enough that air resistance can be ignored.

10. (II) A wet bar of soap slides freely down a ramp 9.0 m long inclined at 8.0°. How long does it take to reach the bottom? Assume $\mu_k = 0.060$.

11. (II) A box is given a push so that it slides across the floor. How far will it go, given that the coefficient of kinetic friction is 0.15 and the push imparts an initial speed of 3.5 m/s?

12. (II) (*a*) Show that the minimum stopping distance for an automobile traveling at speed v is equal to $v^2/2\,\mu_s\,g$, where μ_s is the coefficient of static friction between the tires and the road, and g is the acceleration of gravity. (*b*) What is this distance for a 1200-kg car traveling 95 km/h if $\mu_s = 0.65$? (*c*) What would it be if the car were on the Moon (the acceleration of gravity on the Moon is about $g/6$) but all else stayed the same?

13. (II) A 1280-kg car pulls a 350-kg trailer. The car exerts a horizontal force of 3.6×10^3 N against the ground in order to accelerate. What force does the car exert on the trailer? Assume an effective friction coefficient of 0.15 for the trailer.

14. (II) Police investigators, examining the scene of an accident involving two cars, measure 72-m-long skid marks of one of the cars, which nearly came to a stop before colliding. The coefficient of kinetic friction between rubber and the pavement is about 0.80. Estimate the initial speed of that car assuming a level road.

15. (II) Piles of snow on slippery roofs can become dangerous projectiles as they melt. Consider a chunk of snow at the ridge of a roof with a slope of 34°. (*a*) What is the minimum value of the coefficient of static friction that will keep the snow from sliding down? (*b*) As the snow begins to melt the coefficient of static friction decreases and the snow finally slips. Assuming that the distance from the chunk to the edge of the roof is 6.0 m and the coefficient of kinetic friction is 0.20, calculate the speed of the snow chunk when it slides off the roof. (*c*) If the edge of the roof is 10.0 m above ground, estimate the speed of the snow when it hits the ground.

16. (II) A small box is held in place against a rough vertical wall by someone pushing on it with a force directed upward at 28° above the horizontal. The coefficients of static and kinetic friction between the box and wall are 0.40 and 0.30, respectively. The box slides down unless the applied force has magnitude 23 N. What is the mass of the box?

17. (II) Two crates, of mass 65 kg and 125 kg, are in contact and at rest on a horizontal surface (Fig. 5–32). A 650-N force is exerted on the 65-kg crate. If the coefficient of kinetic friction is 0.18, calculate (*a*) the acceleration of the system, and (*b*) the force that each crate exerts on the other. (*c*) Repeat with the crates reversed.

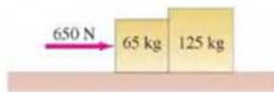


FIGURE 5–32 Problem 17.

18. (II) The crate shown in Fig. 5–33 lies on a plane tilted at an angle $\theta = 25.0°$ to the horizontal, with $\mu_k = 0.19$. (*a*) Determine the acceleration of the crate as it slides down the plane. (*b*) If the crate starts from rest 8.15 m up the plane from its base, what will be the crate's speed when it reaches the bottom of the incline?

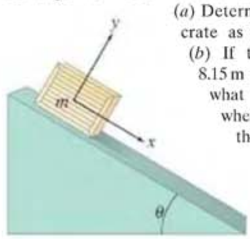


FIGURE 5–33 Crate on inclined plane. Problems 18 and 19.

19. (II) A crate is given an initial speed of 3.0 m/s up the 25.0° plane shown in Fig. 5–33. (*a*) How far up the plane will it go? (*b*) How much time elapses before it returns to its starting point? Assume $\mu_k = 0.17$.

20. (II) Two blocks made of different materials connected together by a thin cord, slide down a plane ramp inclined at an angle θ to the horizontal as shown in Fig. 5–34 (block B is above block A). The masses of the blocks are m_A and m_B, and the coefficients of friction are μ_A and μ_B. If $m_A = m_B = 5.0$ kg, and $\mu_A = 0.20$ and $\mu_B = 0.30$, determine (*a*) the acceleration of the blocks and (*b*) the tension in the cord, for an angle $\theta = 32°$.

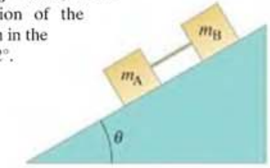


FIGURE 5–34 Problems 20 and 21.

21. (II) For two blocks, connected by a cord and sliding down the incline shown in Fig. 5–34 (see Problem 20), describe the motion (*a*) if $\mu_A < \mu_B$, and (*b*) if $\mu_A > \mu_B$. (*c*) Determine a formula for the acceleration of each block and the tension F_T in the cord in terms of m_A, m_B, and θ; interpret your results in light of your answers to (*a*) and (*b*).

22. (II) A flatbed truck is carrying a heavy crate. The coefficient of static friction between the crate and the bed of the truck is 0.75. What is the maximum rate at which the driver can decelerate and still avoid having the crate slide against the cab of the truck?

23. (II) In Fig. 5–35 the coefficient of static friction between mass m_A and the table is 0.40, whereas the coefficient of kinetic friction is 0.30 (*a*) What minimum value of m_A will keep the system from starting to move? (*b*) What value(s) of m_A will keep the system moving at constant speed?

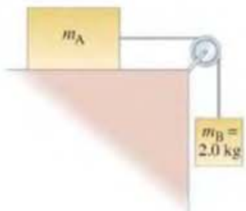


FIGURE 5–35 Problems 23 and 24.

24. (II) Determine a formula for the acceleration of the system shown in Fig. 5–35 in terms of m_A, m_B, and the mass of the cord, m_C. Define any other variables needed.

25. (II) A small block of mass m is given an initial speed v_0 up a ramp inclined at angle θ to the horizontal. It travels a distance d up the ramp and comes to rest. (*a*) Determine a formula for the coefficient of kinetic friction between block and ramp. (*b*) What can you say about the value of the coefficient of static friction?

26. (II) A 75-kg snowboarder has an initial velocity of 5.0 m/s at the top of a 28° incline (Fig. 5–36). After sliding down the 110-m long incline (on which the coefficient of kinetic friction is $\mu_k = 0.18$), the snowboarder has attained a velocity v. The snowboarder then slides along a flat surface (on which $\mu_k = 0.15$) and comes to rest after a distance x. Use Newton's second law to find the snowboarder's acceleration while on the incline and while on the flat surface. Then use these accelerations to determine x.

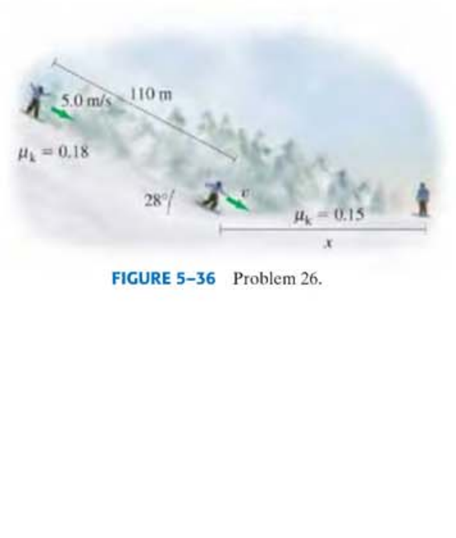


FIGURE 5–36 Problem 26.

27. (II) A package of mass m is dropped vertically onto a horizontal conveyor belt whose speed is $v = 1.5$ m/s, and the coefficient of kinetic friction between the package and the belt is $\mu_k = 0.70$. (*a*) For how much time does the package slide on the belt (until it is at rest relative to the belt)? (*b*) How far does the package move during this time?

28. (II) Two masses $m_A = 2.0$ kg and $m_B = 5.0$ kg are on inclines and are connected together by a string as shown in Fig. 5–37. The coefficient of kinetic friction between each mass and its incline is $\mu_k = 0.30$. If m_A moves up, and m_B moves down, determine their acceleration.

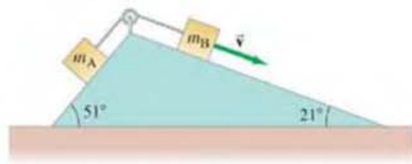


FIGURE 5–37 Problem 28.

29. (II) A child slides down a slide with a 34° incline, and at the bottom her speed is precisely half what it would have been if the slide had been frictionless. Calculate the coefficient of kinetic friction between the slide and the child.

30. (II) (*a*) Suppose the coefficient of kinetic friction between m_A and the plane in Fig. 5–38 is $\mu_k = 0.15$, and that $m_A = m_B = 2.7$ kg. As m_B moves down, determine the magnitude of the acceleration of m_A and m_B, given $\theta = 34°$. (*b*) What smallest value of μ_k will keep the system from accelerating?

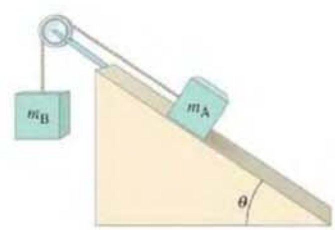


FIGURE 5–38 Problem 30.

31. (III) A 3.0-kg block sits on top of a 5.0-kg block which is on a horizontal surface. The 5.0-kg block is pulled to the right with a force $\vec{F}$ as shown in Fig. 5–39. The coefficient of static friction between all surfaces is 0.60 and the kinetic coefficient is 0.40. (*a*) What is the minimum value of F needed to move the two blocks? (*b*) If the force is 10% greater than your answer for (*a*), what is the acceleration of each block?

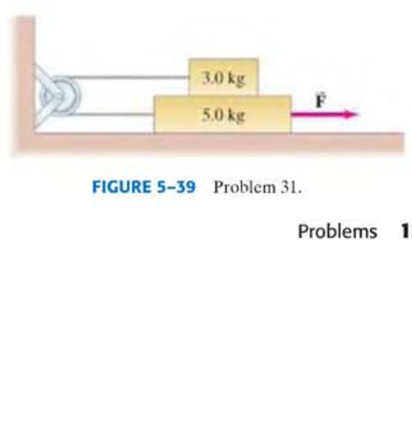


FIGURE 5–39 Problem 31.

32. (III) A 4.0-kg block is stacked on top of a 12.0-kg block, which is accelerating along a horizontal table at $a = 5.2\ \mathrm{m/s^2}$ (Fig. 5–40). Let $\mu_k = \mu_s = \mu$. (*a*) What minimum coefficient of friction μ between the two blocks will prevent the 4.0-kg block from sliding off? (*b*) If μ is only half this minimum value, what is the acceleration of the 4.0-kg block with respect to the table, and (*c*) with respect to the 12.0-kg block? (*d*) What is the force that must be applied to the 12.0-kg block in (*a*) and in (*b*), assuming that the table is frictionless?

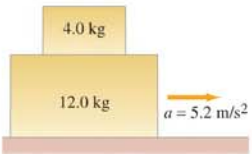


FIGURE 5–40
Problem 32.

33. (III) A small block of mass m rests on the rough, sloping side of a triangular block of mass M which itself rests on a horizontal frictionless table as shown in Fig. 5–41. If the coefficient of static friction is μ, determine the minimum horizontal force F applied to M that will cause the small block m to start moving up the incline.

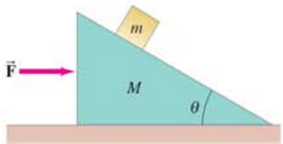


FIGURE 5–41
Problem 33.

5–2 to 5–4 Uniform Circular Motion

34. (I) What is the maximum speed with which a 1200-kg car can round a turn of radius 80.0 m on a flat road if the coefficient of friction between tires and road is 0.65? Is this result independent of the mass of the car?

35. (I) A child sitting 1.20 m from the center of a merry-go-around moves with a speed of 1.30 m/s. Calculate (*a*) the centripetal acceleration of the child and (*b*) the net horizontal force exerted on the child (mass = 22.5 kg).

36. (I) A jet plane traveling 1890 km/h (525 m/s) pulls out of a dive by moving in an arc of radius 4.80 km. What is the plane's acceleration in g's?

37. (II) Is it possible to whirl a bucket of water fast enough in a vertical circle so that the water won't fall out? If so, what is the minimum speed? Define all quantities needed.

38. (II) How fast (in rpm) must a centrifuge rotate if a particle 8.00 cm from the axis of rotation is to experience an acceleration of 125,000 g's?

39. (II) Highway curves are marked with a suggested speed. If this speed is based on what would be safe in wet weather, estimate the radius of curvature for a curve marked 50 km/h. Use Table 5–1.

40. (II) At what minimum speed must a roller coaster be traveling when upside down at the top of a circle (Fig. 5–42) so that the passengers do not fall out? Assume a radius of curvature of 7.6 m.

FIGURE 5–42
Problem 40.

41. (II) A sports car crosses the bottom of a valley with a radius of curvature equal to 95 m. At the very bottom, the normal force on the driver is twice his weight. At what speed was the car traveling?

42. (II) How large must the coefficient of static friction be between the tires and the road if a car is to round a level curve of radius 85 m at a speed of 95 km/h?

43. (II) Suppose the space shuttle is in orbit 400 km from the Earth's surface, and circles the Earth about once every 90 min. Find the centripetal acceleration of the space shuttle in its orbit. Express your answer in terms of g, the gravitational acceleration at the Earth's surface.

44. (II) A bucket of mass 2.00 kg is whirled in a vertical circle of radius 1.10 m. At the lowest point of its motion the tension in the rope supporting the bucket is 25.0 N. (*a*) Find the speed of the bucket. (*b*) How fast must the bucket move at the top of the circle so that the rope does not go slack?

45. (II) How many revolutions per minute would a 22-m-diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point?

46. (II) Use dimensional analysis (Section 1–7) to obtain the form for the centripetal acceleration, $a_R = v^2/r$.

47. (II) A jet pilot takes his aircraft in a vertical loop (Fig. 5–43). (*a*) If the jet is moving at a speed of 1200 km/h at the lowest point of the loop, determine the minimum radius of the circle so that the centripetal acceleration at the lowest point does not exceed 6.0 g's. (*b*) Calculate the 78-kg pilot's effective weight (the force with which the seat pushes up on him) at the bottom of the circle, and (*c*) at the top of the circle (assume the same speed).

FIGURE 5–43
Problem 47.

48. (II) A proposed space station consists of a circular tube that will rotate about its center (like a tubular bicycle tire), Fig. 5–44. The circle formed by the tube has a diameter of about 1.1 km. What must be the rotation speed (revolutions per day) if an effect equal to gravity at the surface of the Earth (1.0 g) is to be felt?

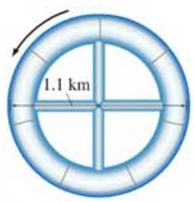


FIGURE 5–44
Problem 48.

49. (II) On an ice rink two skaters of equal mass grab hands and spin in a mutual circle once every 2.5 s. If we assume their arms are each 0.80 m long and their individual masses are 60.0 kg, how hard are they pulling on one another?

50. (II) Redo Example 5–11, precisely this time, by not ignoring the weight of the ball which revolves on a string 0.600 m long. In particular, find the magnitude of $\vec{\mathbf{F}}_T$, and the angle it makes with the horizontal. [*Hint*: Set the horizontal component of $\vec{\mathbf{F}}_T$ equal to ma_R; also, since there is no vertical motion, what can you say about the vertical component of $\vec{\mathbf{F}}_T$?]

51. (II) A coin is placed 12.0 cm from the axis of a rotating turntable of variable speed. When the speed of the turntable is slowly increased, the coin remains fixed on the turntable until a rate of 35.0 rpm (revolutions per minute) is reached, at which point the coin slides off. What is the coefficient of static friction between the coin and the turntable?

52. (II) The design of a new road includes a straight stretch that is horizontal and flat but that suddenly dips down a steep hill at 22°. The transition should be rounded with what minimum radius so that cars traveling 95 km/h will not leave the road (Fig. 5–45)?

FIGURE 5–45
Problem 52.

53. (II) A 975-kg sports car (including driver) crosses the rounded top of a hill (radius = 88.0 m) at 12.0 m/s. Determine (*a*) the normal force exerted by the road on the car, (*b*) the normal force exerted by the car on the 72.0-kg driver, and (*c*) the car speed at which the normal force on the driver equals zero.

54. (II) Two blocks, with masses m_A and m_B, are connected to each other and to a central post by cords as shown in Fig. 5–46. They rotate about the post at frequency f (revolutions per second) on a frictionless horizontal surface at distances r_A and r_B from the post. Derive an algebraic expression for the tension in each segment of the cord (assumed massless).

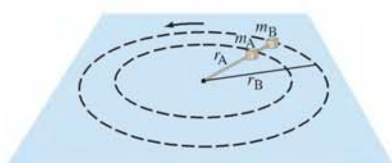


FIGURE 5–46 Problem 54.

55. (II) Tarzan plans to cross a gorge by swinging in an arc from a hanging vine (Fig. 5–47). If his arms are capable of exerting a force of 1350 N on the rope, what is the maximum speed he can tolerate at the lowest point of his swing? His mass is 78 kg and the vine is 5.2 m long.

FIGURE 5–47
Problem 55.

56. (II) A pilot performs an evasive maneuver by diving vertically at 310 m/s. If he can withstand an acceleration of 9.0 g's without blacking out, at what altitude must he begin to pull out of the dive to avoid crashing into the sea?

57. (III) The position of a particle moving in the *xy* plane is given by $\vec{\mathbf{r}} = 2.0\cos(3.0\,\text{rad/s}\;t)\,\hat{\mathbf{i}} + 2.0\sin(3.0\,\text{rad/s}\;t)\,\hat{\mathbf{j}}$, where r is in meters and t is in seconds. (*a*) Show that this represents circular motion of radius 2.0 m centered at the origin. (*b*) Determine the velocity and acceleration vectors as functions of time. (*c*) Determine the speed and magnitude of the acceleration. (*d*) Show that $a = v^2/r$. (*e*) Show that the acceleration vector always points toward the center of the circle.

58. (III) If a curve with a radius of 85 m is properly banked for a car traveling 65 km/h, what must be the coefficient of static friction for a car not to skid when traveling at 95 km/h?

59. (III) A curve of radius 68 m is banked for a design speed of 85 km/h. If the coefficient of static friction is 0.30 (wet pavement), at what range of speeds can a car safely make the curve? [*Hint*: Consider the direction of the friction force when the car goes too slow or too fast.]

*5–5 Nonuniform Circular Motion

***60.** (II) A particle starting from rest revolves with uniformly increasing speed in a clockwise circle in the *xy* plane. The center of the circle is at the origin of an *xy* coordinate system. At $t = 0$, the particle is at $x = 0.0$, $y = 2.0$ m. At $t = 2.0$ s, it has made one-quarter of a revolution and is at $x = 2.0$ m, $y = 0.0$. Determine (*a*) its speed at $t = 2.0$ s, (*b*) the average velocity vector, and (*c*) the average acceleration vector during this interval.

***61.** (II) In Problem 60 assume the tangential acceleration is constant and determine the components of the instantaneous acceleration at (*a*) $t = 0.0$, (*b*) $t = 1.0$ s, and (*c*) $t = 2.0$ s.

***62.** (II) An object moves in a circle of radius 22 m with its speed given by $v = 3.6 + 1.5t^2$, with v in meters per second and t in seconds. At $t = 3.0$ s, find (*a*) the tangential acceleration and (*b*) the radial acceleration.

***63.** (III) A particle rotates in a circle of radius 3.80 m. At a particular instant its acceleration is 1.15 m/s² in a direction that makes an angle of 38.0° to its direction of motion. Determine its speed (*a*) at this moment and (*b*) 2.00 s later, assuming constant tangential acceleration.

***64.** (III) An object of mass m is constrained to move in a circle of radius r. Its tangential acceleration as a function of time is given by $a_{\tan} = b + ct^2$, where b and c are constants. If $v = v_0$ at $t = 0$, determine the tangential and radial components of the force, $F_{\tan}$ and F_R, acting on the object at any time $t > 0$.

*5–6 Velocity-Dependent Forces

***65.** (I) Use dimensional analysis (Section 1–7) in Example 5–17 to determine if the time constant τ is $\tau = m/b$ or $\tau = b/m$.

***66.** (II) The terminal velocity of a 3×10^{-5} kg raindrop is about 9 m/s. Assuming a drag force $F_D = -bv$, determine (*a*) the value of the constant b and (*b*) the time required for such a drop, starting from rest, to reach 63% of terminal velocity.

***67.** (II) An object moving vertically has $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at $t = 0$. Determine a formula for its velocity as a function of time assuming a resistive force $F = -bv$ as well as gravity for two cases: (*a*) $\vec{\mathbf{v}}_0$ is downward and (*b*) $\vec{\mathbf{v}}_0$ is upward.

*68. (III) The drag force on large objects such as cars, planes, and sky divers moving through air is more nearly $F_D = -bv^2$. (*a*) For this quadratic dependence on v, determine a formula for the terminal velocity v_T, of a vertically falling object. (*b*) A 75-kg sky diver has a terminal velocity of about 60 m/s; determine the value of the constant b. (*c*) Sketch a curve like that of Fig. 5–27b for this case of $F_D \propto v^2$. For the same terminal velocity, would this curve lie above or below that in Fig. 5–27? Explain why.

*69. (III) A bicyclist can coast down a 7.0° hill at a steady 9.5 km/h. If the drag force is proportional to the square of the speed v, so that $F_D = -cv^2$, calculate (*a*) the value of the constant c and (*b*) the average force that must be applied in order to descend the hill at 25 km/h. The mass of the cyclist plus bicycle is 80.0 kg. Ignore other types of friction.

*70. (III) Two drag forces act on a bicycle and rider: F_{D1} due to rolling resistance, which is essentially velocity independent; and F_{D2} due to air resistance, which is proportional to v^2. For a specific bike plus rider of total mass 78 kg, $F_{D1} \approx 4.0$ N; and for a speed of 2.2 m/s, $F_{D2} \approx 1.0$ N. (*a*) Show that the total drag force is

$$F_D = 4.0 + 0.21v^2,$$

where v is in m/s, and F_D is in N and opposes the motion. (*b*) Determine at what slope angle θ the bike and rider can coast downhill at a constant speed of 8.0 m/s.

*71. (III) Determine a formula for the position and acceleration of a falling object as a function of time if the object starts from rest at $t = 0$ and undergoes a resistive force $F = -bv$, as in Example 5–17.

*72. (III) A block of mass m slides along a horizontal surface lubricated with a thick oil which provides a drag force proportional to the square root of velocity:

$$F_D = -bv^{\frac{1}{2}}.$$

If $v = v_0$ at $t = 0$, determine v and x as functions of time.

*73. (III) Show that the maximum distance the block in Problem 72 can travel is $2m\,v_0^{3/2}/3b$.

*74. (III) You dive straight down into a pool of water. You hit the water with a speed of 5.0 m/s, and your mass is 75 kg. Assuming a drag force of the form $F_D = -(1.00 \times 10^4\ \text{kg/s})\,v$, how long does it take you to reach 2% of your original speed? (Ignore any effects of buoyancy.)

*75. (III) A motorboat traveling at a speed of 2.4 m/s shuts off its engines at $t = 0$. How far does it travel before coming to rest if it is noted that after 3.0 s its speed has dropped to half its original value? Assume that the drag force of the water is proportional to v.

General Problems

76. A coffee cup on the horizontal dashboard of a car slides forward when the driver decelerates from 45 km/h to rest in 3.5 s or less, but not if she decelerates in a longer time. What is the coefficient of static friction between the cup and the dash? Assume the road and the dashboard are level (horizontal).

77. A 2.0-kg silverware drawer does not slide readily. The owner gradually pulls with more and more force, and when the applied force reaches 9.0 N, the drawer suddenly opens, throwing all the utensils to the floor. What is the coefficient of static friction between the drawer and the cabinet?

78. A roller coaster reaches the top of the steepest hill with a speed of 6.0 km/h. It then descends the hill, which is at an average angle of 45° and is 45.0 m long. What will its speed be when it reaches the bottom? Assume $\mu_k = 0.12$.

79. An 18.0-kg box is released on a 37.0° incline and accelerates down the incline at 0.220 m/s². Find the friction force impeding its motion. How large is the coefficient of friction?

80. A flat puck (mass M) is revolved in a circle on a frictionless air hockey table top, and is held in this orbit by a light cord which is connected to a dangling mass (mass m) through a central hole as shown in Fig. 5–48. Show that the speed of the puck is given by $v = \sqrt{mgR/M}$.

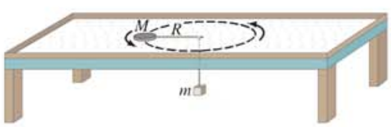


FIGURE 5–48 Problem 80.

81. A motorcyclist is coasting with the engine off at a steady speed of 20.0 m/s but enters a sandy stretch where the coefficient of kinetic friction is 0.70. Will the cyclist emerge from the sandy stretch without having to start the engine if the sand lasts for 15 m? If so, what will be the speed upon emerging?

82. In a "Rotor-ride" at a carnival, people rotate in a vertical cylindrically walled "room." (See Fig. 5–49). If the room radius was 5.5 m, and the rotation frequency 0.50 revolutions per second when the floor drops out, what minimum coefficient of static friction keeps the people from slipping down? People on this ride said they were "pressed against the wall." Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides nausea)? [*Hint:* Draw a free-body diagram for a person.]

FIGURE 5–49 Problem 82.

83. A device for training astronauts and jet fighter pilots is designed to rotate the trainee in a horizontal circle of radius 11.0 m. If the force felt by the trainee is 7.45 times her own weight, how fast is she rotating? Express your answer in both m/s and rev/s.

84. A 1250-kg car rounds a curve of radius 72 m banked at an angle of 14°. If the car is traveling at 85 km/h, will a friction force be required? If so, how much and in what direction?

85. Determine the tangential and centripetal components of the net force exerted on a car (by the ground) when its speed is 27 m/s, and it has accelerated to this speed from rest in 9.0 s on a curve of radius 450 m. The car's mass is 1150 kg.

86. The 70.0-kg climber in Fig. 5–50 is supported in the "chimney" by the friction forces exerted on his shoes and back. The static coefficients of friction between his shoes and the wall, and between his back and the wall, are 0.80 and 0.60, respectively. What is the minimum normal force he must exert? Assume the walls are vertical and that the static friction forces are both at their maximum. Ignore his grip on the rope.

FIGURE 5–50 Problem 86.

87. A small mass m is set on the surface of a sphere, Fig. 5–51. If the coefficient of static friction is $\mu_s = 0.70$, at what angle ϕ would the mass start sliding?

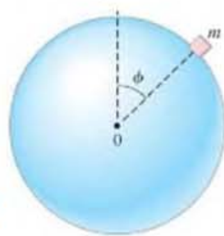


FIGURE 5–51 Problem 87.

88. A 28.0-kg block is connected to an empty 2.00-kg bucket by a cord running over a frictionless pulley (Fig. 5–52). The coefficient of static friction between the table and the block is 0.45 and the coefficient of kinetic friction between the table and the block is 0.32. Sand is gradually added to the bucket until the system just begins to move. (*a*) Calculate the mass of sand added to the bucket. (*b*) Calculate the acceleration of the system.

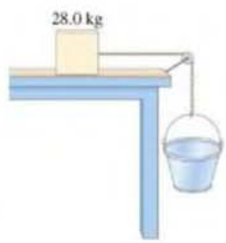


FIGURE 5–52 Problem 88.

89. A car is heading down a slippery road at a speed of 95 km/h. The minimum distance within which it can stop without skidding is 66 m. What is the sharpest curve the car can negotiate on the icy surface at the same speed without skidding?

90. What is the acceleration experienced by the tip of the 1.5-cm-long sweep second hand on your wrist watch?

91. An airplane traveling at 480 km/h needs to reverse its course. The pilot decides to accomplish this by banking the wings at an angle of 38°. (*a*) Find the time needed to reverse course. (*b*) Describe any additional force the passengers experience during the turn. [*Hint:* Assume an aerodynamic "lift" force that acts perpendicularly to the flat wings; see Fig. 5–53.]

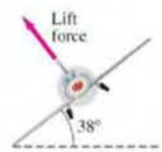


FIGURE 5–53 Problem 91.

92. A banked curve of radius R in a new highway is designed so that a car traveling at speed v_0 can negotiate the turn safely on glare ice (zero friction). If a car travels too slowly then it will slip toward the center of the circle. If it travels too fast, it will slip away from the center of the circle. If the coefficient of static friction increases, it becomes possible for a car to stay on the road while traveling at a speed within a range from v_{min} to v_{max}. Derive formulas for v_{min} and v_{max} as functions of μ_s, v_0, and R.

93. A small bead of mass m is constrained to slide without friction inside a circular vertical hoop of radius r which rotates about a vertical axis (Fig. 5–54) at a frequency f. (*a*) Determine the angle θ where the bead will be in equilibrium—that is, where it will have no tendency to move up or down along the hoop. (*b*) If $f = 2.00$ rev/s and $r = 22.0$ cm, what is θ? (*c*) Can the bead ride as high as the center of the circle ($\theta = 90°$)? Explain.

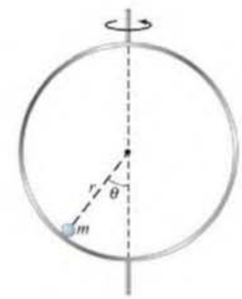


FIGURE 5–54 Problem 93.

94. *Earth is not quite an inertial frame.* We often make measurements in a reference frame fixed on the Earth, assuming Earth is an inertial reference frame. But the Earth rotates, so this assumption is not quite valid. Show that this assumption is off by 3 parts in 1000 by calculating the acceleration of an object at Earth's equator due to Earth's daily rotation, and compare to $g = 9.80\ m/s^2$, the acceleration due to gravity.

95. While fishing, you get bored and start to swing a sinker weight around in a circle below you on a 0.45-m piece of fishing line. The weight makes a complete circle every 0.50 s. What is the angle that the fishing line makes with the vertical? [*Hint:* See Fig. 5–20.]

96. Consider a train that rounds a curve with a radius of 570 m at a speed of 160 km/h (approximately 100 mi/h). (*a*) Calculate the friction force needed on a train passenger of mass 75 kg if the track is not banked and the train does not tilt. (*b*) Calculate the friction force on the passenger if the train tilts at an angle of 8.0° toward the center of the curve.

97. A car starts rolling down a 1-in-4 hill (1-in-4 means that for each 4 m traveled along the road, the elevation change is 1 m). How fast is it going when it reaches the bottom after traveling 55 m? (*a*) Ignore friction. (*b*) Assume an effective coefficient of friction equal to 0.10.

98. The sides of a cone make an angle ϕ with the vertical. A small mass m is placed on the inside of the cone and the cone, with its point down, is revolved at a frequency f (revolutions per second) about its symmetry axis. If the coefficient of static friction is μ_s, at what positions on the cone can the mass be placed without sliding on the cone? (Give the maximum and minimum distances, r, from the axis).

99. A 72-kg water skier is being accelerated by a ski boat on a flat ("glassy") lake. The coefficient of kinetic friction between the skier's skis and the water surface is $\mu_k = 0.25$ (Fig. 5–55). (*a*) What is the skier's acceleration if the rope pulling the skier behind the boat applies a horizontal tension force of magnitude $F_T = 240$ N to the skier ($\theta = 0°$)? (*b*) What is the skier's horizontal acceleration if the rope pulling the skier exerts a force of $F_T = 240$ N on the skier at an upward angle $\theta = 12°$? (*c*) Explain why the skier's acceleration in part (*b*) is greater than that in part (*a*).

FIGURE 5–55 Problem 99.

100. A ball of mass $m = 1.0$ kg at the end of a thin cord of length $r = 0.80$ m revolves in a vertical circle about point O, as shown in Fig. 5–56. During the time we observe it, the only forces acting on the ball are gravity and the tension in the cord. The motion is circular but not uniform because of the force of gravity. The ball increases in speed as it descends and decelerates as it rises on the other side of the circle. At the moment the cord makes an angle $\theta = 30°$ below the horizontal, the ball's speed is 6.0 m/s. At this point, determine the tangential acceleration, the radial acceleration, and the tension in the cord, F_T. Take θ increasing downward as shown.

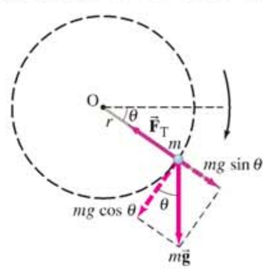


FIGURE 5–56 Problem 100.

101. A car drives at a constant speed around a banked circular track with a diameter of 127 m. The motion of the car can be described in a coordinate system with its origin at the center of the circle. At a particular instant the car's acceleration in the horizontal plane is given by

$$\vec{a} = (-15.7\hat{i} - 23.2\hat{j})\ \text{m/s}^2.$$

(*a*) What is the car's speed? (*b*) Where (x and y) is the car at this instant?

*Numerical/Computer

*102. (III) The force of air resistance (drag force) on a rapidly falling body such as a skydiver has the form $F_D = -kv^2$, so that Newton's second law applied to such an object is

$$m\frac{dv}{dt} = mg - kv^2,$$

where the downward direction is taken to be positive. (*a*) Use numerical integration [Section 2–9] to estimate (within 2%) the position, speed, and acceleraton, from $t = 0$ up to $t = 15.0$ s, for a 75-kg skydiver who starts from rest, assuming $k = 0.22$ kg/m. (*b*) Show that the diver eventually reaches a steady speed, the *terminal speed*, and explain why this happens. (*c*) How long does it take for the skydiver to reach 99.5% of the terminal speed?

*103. (III) The coefficient of kinetic friction μ_k between two surfaces is not strictly independent of the velocity of the object. A possible expression for μ_k for wood on wood is

$$\mu_k = \frac{0.20}{\left(1 + 0.0020v^2\right)^2},$$

where v is in m/s. A wooden block of mass 8.0 kg is at rest on a wooden floor, and a constant horizontal force of 41 N acts on the block. Use numerical integration [Section 2–9] to determine and graph (*a*) the speed of the block, and (*b*) its position, as a function of time from 0 to 5.0 s. (*c*) Determine the percent difference for the speed and position at 5.0 s if μ_k is constant and equal to 0.20.

*104. (III) Assume a net force $F = -mg - kv^2$ acts during the upward vertical motion of a 250-kg rocket, starting at the moment ($t = 0$) when the fuel has burned out and the rocket has an upward speed of 120 m/s. Let $k = 0.65$ kg/m. Estimate v and y at 1.0-s intervals for the upward motion only, and estimate the maximum height reached. Compare to free-flight conditions without air resistance ($k = 0$).

Answers to Exercises

A: (*c*).

B: F_{Px} is insufficient to keep the box moving for long.

C: No—the acceleration is not constant (in direction).

D: (*a*), it doubles.

E: (*d*).

F: (*a*).

G: (*c*).

H: Yes.

I: (*a*) No change; (*b*) 4 times larger.